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A TEXTBOOK
ON
SURVEYING AND MAPPING

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INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

ANSWERS TO QUESTIONS

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NOTICE.

There is a break in the continuity of the question numbers, etc., between the section on Formulas and the section on Geometry and Trigonometry, and between Geometry and Trigonometry and Surveying. This does not affect the subject-matter, which is published in full, and in regular order.

507201

A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

CONTAINED IN THE

EXAMINATION QUESTIONS

The Keys that follow have been divided into sections corresponding to the Examination Questions to which they refer. The answers and solutions have been numbered to correspond with the questions. When the answer to a question involves a repetition of statements given in the Instruction Paper, the reader has been referred to a numbered article, the reading of which will enable him to answer the question himself.

To be of the greatest benefit, the Keys should be used sparingly. They should be used much in the same manner as a pupil would go to a teacher for instruction with regard to answering some example he was unable to solve. If used in this manner, the Keys will be of great help and assistance to the student, and will be a source of encouragement to him in studying the various papers composing the Course.

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ARITHMETIC.

(SECTIONS 1–3.)

(QUESTIONS 1–75.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.

605 = Six hundred five.

28,284 = Twenty-eight thousand, two hundred eighty-four,

9,006,042 = Nine million, six thousand and forty-two.

850,317,002 = Eight hundred fifty million, three hundred seventeen thousand and two.

700,004 = Seven hundred thousand and four.

(6) Seven thousand six hundred = 7,600.

Eighty-one thousand four hundred two = 81,402.

Five million, four thousand and seven = 5,004,007.

One hundred and eight million, ten thousand and one = 108,010,001.

Eighteen million and six = 18,000,006.

Thirty thousand and ten = 30,010.

(7) In adding whole numbers, place the numbers to be added directly under each other so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column, and reserve 1 ten for the column

$$\begin{array}{r}
3290\\
504\\
865403\\
2074\\
81\\
7\\
\hline
871359
\end{array}\quad \text{Ans.}$$

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of tens. $1+8+7+9=25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $2+4+5+2=13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $1+2+5+3=11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten-thousand for the column of ten-thousands. $1+6=7$ ten-thousands. Place this seven ten-thousands in the ten-thousands column. There is but one figure 8 in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

A simpler (though less scientific) explanation of the same problem is the following: $7+1+4+3+4+0=19$; write the nine and reserve the 1. $1+8+7+0+0+9=25$; write the 5 and reserve the 2. $2+0+4+5+2=13$; write the 3 and reserve the 1. $1+2+5+3=11$; write the 1 and reserve 1. $1+6=7$; write the 7. Bring down the 8 to its place in the sum.

(8)

$$\begin{array}{r l}
709 & \\
8304725 & \\
391 & \\
100302 & \\
300 & \\
909 & \\
\hline
8407336 & \text{Ans.}
\end{array}$$

(9) (*a*) In subtracting whole numbers, place the subtrahend or smaller number under the minuend or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract. We can not subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. As 1 *ten* = 10 *units*, we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so

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only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We can not subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$\begin{array}{r}50962\\ 3338\\ \hline 47624\end{array}\text{ Ans.}$$

$$(b)\quad\begin{array}{r}15339\\ 10001\\ \hline 5338\end{array}\text{ Ans.}$$

(10) $(a)\quad\begin{array}{r}70968\\ 32975\\ \hline 37993\end{array}\text{ Ans.}\qquad(b)\quad\begin{array}{r}100000\\ 98735\\ \hline 1265\end{array}\text{ Ans.}$

(11) We have given the minuend or greater number (1,004) and the difference or remainder (49). Placing these in the usual form of subtraction we have $\begin{array}{r}1004\\ \text{——}\\ \hline 49\end{array}$ in which the dash (——) represents the number sought. This number is evidently *less* than 1,004 by the difference 49, hence, 1,004 − 49 = 955, the smaller number. For the sum of the two numbers we then have

$$\begin{array}{rl}1004 & \textit{larger}\\ 955 & \textit{smaller}\\ \hline 1959 & \textit{sum.}\ \text{Ans.}\end{array}$$

Or, this problem may be solved as follows: If the greater of two numbers is 1,004, and the difference between them is 49, then it is evident that the smaller number must be equal to the difference between the greater number (1,004) and the difference (49); or, 1,004 − 49 = 955, the smaller number. Since the greater number equals 1,004 and the smaller number equals 955, their sum equals 1,004 + 955 = 1,959 sum. Ans.

(12) The numbers connected by the plus (+) sign must first be added. Performing these operations we have

$$\begin{array}{r l} 5962 & \\ 8471 & \\ 9023 & \\ \hline 23456 & \textit{sum.} \end{array} \qquad \begin{array}{r l} 3874 & \\ 2039 & \\ \hline 5913 & \textit{sum.} \end{array}$$

Subtracting the smaller number (5,913) from the greater (23,456) we have

$$\begin{array}{r l} 23456 & \\ 5913 & \\ \hline 17543 & \textit{difference.} \quad \text{Ans.} \end{array}$$

(13) \$44675 = amount willed to his son.
26380 = amount willed to his daughter.
\$71055 = amount willed to his two children.
\$125000 = amount willed to his wife and two children.
71055 = amount willed to his two children.
\$53945 = amount willed to his wife. Ans.

(14) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*)

$$\begin{array}{r l} 526387 & \\ 7 & \\ \hline 3684709 & \text{Ans.} \end{array}$$

7 × 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 tens + 4 tens reserved = 60 tens or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve

the 2 thousands. 7 × 6 thousands = 42 thousands; 42 + 2 thousands reserved = 44 thousands or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten-thousands. 7 × 2 ten-thousands = 14 ten-thousands; 14 + 4 ten-thousands reserved = 18 ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7 × 5 hundred-thousands = 35 hundred-thousands; 35 + 1 hundred-thousand reserved = 36 hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. 7 × 6 = 42; 42 + 2 reserved = 44; write the 4 and reserve 4. 7 × 2 = 14; 14 + 4 reserved = 18; write the 8 and reserve the 1. 7 × 5 = 35; 35 + 1 reserved = 36; write the 36.

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, 7 × 700,298 and 10 × 700,298. The actual operation is performed as follows:

(*b*)

$$\begin{array}{r} 700298 \\ \underline{17} \\ 4902086 \\ \underline{700298} \\ 11905066 \end{array} \quad \text{Ans.}$$

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we say 1 times 700298 = 700298, and write 700298 under the first partial product, as shown, with the right-hand figure 8 under the multiplier 1. Add the two partial products; their sum equals the entire product.

(*c*) Multiply any two of the numbers together and multiply their product by the third number.

217
103
651
2170
22351
67
156457
134106
1497517 Ans.

(15) If your watch ticks every second, then to find how many times it ticks in one week it is necessary to find the number of seconds in 1 week.

60 seconds = 1 minute.
60 minutes = 1 hour.
3600 seconds = 1 hour.
24 hours = 1 day.
14400
7200
86400 seconds = 1 day.
7 days = 1 week.
604800 seconds in 1 week or the number of times that your watch ticks in 1 week.
Ans.

(16) If a monthly publication contains 24 pages, a yearly volume will contain 12×24 or 288 pages, since there are 12 months in one year; and eight yearly volumes will contain 8×288, or 2,304 pages.

24
12
288
8
2304 Ans.

(17) If an engine and boiler are worth $3,246, and the building is worth 3 times as much, plus $1,200, then the building is worth

$3246
3
9738
plus 1200
$10938 = value of building.

If the tools are worth twice as much as the building, plus \$1,875, then the tools are worth

$$\begin{array}{rr} & \$10938 \\ & 2 \\ \hline & 21876 \\ \text{plus} & 1875 \\ \hline & \$23751 \end{array} = \text{value of tools.}$$

$$\begin{array}{lr} \text{Value of building} = & \$10938 \\ \text{Value of tools} \quad = & 23751 \\ \hline & \$34689 \end{array} = \text{value of the building and tools. (a) Ans.}$$

$$\begin{array}{lr} \text{Value of engine and boiler} = & \$\ 3246 \\ \text{Value of building and tools} = & 34689 \\ \hline & \$37935 \end{array} = \text{value of the whole plant. (b) Ans.}$$

(18) (*a*) $(72 \times 48 \times 28 \times 5) \div (96 \times 15 \times 7 \times 6)$.

Placing the numerator over the denominator the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$$

The 5 in the *dividend* and 15 in the *divisor* are both *divisible* by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. *Cross off* the 5 and write the 1 *over* it; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \overset{1}{\cancel{5}}}{96 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

The 5 and 15 are *not* to be considered any longer, and, in fact, may be erased entirely and the 1 and 3 placed in their stead, and treated as if the 5 and 15 *never* existed. Thus,

$$\frac{72 \times 48 \times 28 \times 1}{96 \times 3 \times 7 \times 6} =$$

72 in the *dividend* and 96 in the *divisor* are *divisible* by 12, since 72 divided by 12 equals 6, and 96 divided by 12 equals 8. *Cross off* the 72 and write the 6 *over* it: also, *cross off* the 96 and write the 8 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times 1}{\underset{8}{\cancel{96}} \times 3 \times 7 \times 6} =$$

The 72 and 96 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 6 and 8 placed in their stead, and treated as if the 72 and 96 *never* existed. Thus,

$$\frac{6 \times 48 \times 28 \times 1}{8 \times 3 \times 7 \times 6} =$$

Again, 28 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{6 \times 48 \times \overset{4}{\cancel{28}} \times 1}{8 \times 3 \times \underset{1}{\cancel{7}} \times 6} =$$

The 28 and 7 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 4 and 1 placed in their stead, and treated as if the 28 and 7 *never* existed. Thus,

$$\frac{6 \times 48 \times 4 \times 1}{8 \times 3 \times 1 \times 6} =$$

Again, 48 in the *dividend* and 6 in the *divisor* are *divisible* by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, cross off the 6 and write the 1 *under* it. Thus,

$$\frac{6 \times \overset{8}{\cancel{48}} \times 4 \times 1}{8 \times 3 \times 1 \times \underset{1}{\cancel{6}}} =$$

The 48 and 6 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 8 and 1 placed in their stead, and treated as if the 48 and 6 *never* existed. Thus,

$$\frac{6\times 8\times 4\times 1}{8\times 3\times 1\times 1}=$$

Again, 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, cross off the 3 and write the 1 *under* it. Thus,

$$\frac{\overset{2}{\cancel{6}}\times 8\times 4\times 1}{8\times \underset{1}{\cancel{3}}\times 1\times 1}=$$

The 6 and 3 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{2\times 8\times 4\times 1}{8\times 1\times 1\times 1}=$$

Canceling the 8 in the dividend and the 8 in the divisor, the result is

$$\frac{2\times \overset{1}{\cancel{8}}\times 4\times 1}{\underset{1}{\cancel{8}}\times 1\times 1\times 1}=\frac{2\times 1\times 4\times 1}{1\times 1\times 1\times 1}.$$

Since there are *no two remaining numbers* (one in the dividend and one in the divisor) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the *dividend* together, and divide their *product* by the *product* of all the *uncanceled* numbers in the divisor. The *result* will be the *quotient*. The *product* of all the *uncanceled numbers* in the *dividend* equals $2\times 1\times 4\times 1=8$; the product of all the *uncanceled* numbers in the *divisor* equals $1\times 1\times 1\times 1=1$.

Hence, $$\frac{2\times 1\times 4\times 1}{1\times 1\times 1\times 1}=\frac{8}{1}=8.\quad \text{Ans.}$$

Or, $$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}}\times \overset{\cancel{8}}{\cancel{48}}\times \overset{4}{\cancel{28}}\times \overset{1}{\cancel{5}}}{\underset{\underset{1}{\cancel{8}}}{\cancel{96}}\times \underset{\underset{1}{\cancel{3}}}{\cancel{15}}\times \cancel{7}\times \underset{1}{\cancel{6}}}=\frac{8}{1}=8.\quad \text{Ans.}$$

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the *dividend* and 70 in the *divisor* are both *divisible* by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

The 50 and 70 are not to be considered any longer, and, in fact, may be erased entirely and the 5 and 7 placed in their stead, and treated as if the 50 and 70 *never* existed. Thus,

$$\frac{80 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 20} =$$

Also, 80 in the *dividend* and 20 in the *divisor* are *divisible* by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. Cross off the 80 and write the 4 *over* it; also, cross off the 20 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times \underset{1}{\cancel{20}}} =$$

The 80 and 20 are *not* to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 80 and 20 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 1} =$$

Again, 16 in the *dividend* and 24 in the *divisor* are *divisible* by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also cross off the 24 and write the 3 *under* it. Thus,

$$\frac{4 \times 60 \times 5 \times \overset{2}{\cancel{16}} \times 14}{7 \times 50 \times \underset{3}{\cancel{24}} \times 1} =$$

The 16 and 24 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 3 placed in their stead, and treated as if the 16 and 24 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 2 \times 14}{7 \times 50 \times 3 \times 1} =$$

Again, 60 in the *dividend* and 50 in the *divisor* are *divisible* by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{4 \times \overset{6}{\cancel{60}} \times 5 \times 2 \times 14}{7 \times \underset{5}{\cancel{50}} \times 3 \times 1} =$$

The 60 and 50 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 5 placed in their stead, and treated as if the 60 and 50 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 14}{7 \times 5 \times 3 \times 1} =$$

The 14 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times 5 \times 3 \times 1} =$$

The 14 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 14 and 7 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 2}{1 \times 5 \times 3 \times 1} =$$

The 5 in the *dividend* and 5 in the *divisor* are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the 5 of the *dividend* and write the 1 *over* it; also, cross off the 5 of the *divisor* and write the 1 *under* it. Thus,

$$\frac{4 \times 6 \times \overset{1}{\cancel{5}} \times 2 \times 2}{1 \times \underset{1}{\cancel{5}} \times 3 \times 1} =$$

The 5 in the *dividend* and 5 in the *divisor* are not to be considered any longer, and, in fact, may be erased entirely and 1 and 1 placed in their stead, and treated as if the 5 and 5 *never* existed. Thus,

$$\frac{4 \times 6 \times 1 \times 2 \times 2}{1 \times 1 \times 3 \times 1} =$$

The 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3 and place 1 *under* it. Thus,

$$\frac{4 \times \overset{2}{\cancel{6}} \times 1 \times 2 \times 2}{1 \times 1 \times \underset{1}{\cancel{3}} \times 1} =$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{4 \times 2 \times 1 \times 2 \times 2}{1 \times 1 \times 1 \times 1} = \frac{32}{1} = 32. \quad \text{Ans.}$$

$$\text{Hence, } \frac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{\underset{1}{\cancel{5}}}{\cancel{50}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{24}} \times \underset{1}{\cancel{20}}} = \frac{4 \times 2 \times 1 \times 2 \times 2}{1 \times 1 \times 1 \times 1} = \frac{32}{1} = 32. \quad \text{Ans.}$$

(19) 28 acres of land at $133 an acre would cost 28 × $133 = $3,724.

$$\begin{array}{r} 28 \\ \hline 1064 \\ 266 \\ \hline \$3724 \end{array}$$

If a mechanic earns $1,500 a year and his expenses are $968 per year, then he would save $1500—$968, or $532 per year.

968

$532

If he saves $532 in 1 year, to save $3,724 it would take as many years as $532 is contained times in $3,724, or 7 years.

532) 3724 (7 years. Ans.

3724

(20) If the freight train ran 365 miles in one week, and 3 times as far lacking 246 miles the next week, then it ran (3 × 365 miles) – 246 miles, or 849 miles the second week. Thus,

365

3

1095

246

difference 849 miles. Ans.

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in one mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, then in going from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

16)1869120(116820 rev. Ans

16

26

16

109

96

131

128

32

32

0

(22) (*a*) 576)589824(1024 **Ans.**
576
1382
1152
2304
2304

(*b*) 43911)369730620(8420 **Ans.**
351288
184426
175644
87822
87822
0

(*c*) 505)2527525(5005 **Ans.**
2525
2525
2525

(*d*) 1234)4961794302(4020903 **Ans.**
4936
2579
2468
11143
11106
3702
3702

(23) The harness evidently cost the difference between $444 and the amount which he paid for the horse and wagon.

Since $264 + $153 = $417, the amount paid for the horse and wagon, $444 − $417 = $27, the cost of the harness.

$264
153
$417

$444
417
$27 **Ans.**

(24) (*a*)

$$\begin{array}{r}
1024\\
576\\
\hline
6144\\
7168\\
5120\\
\hline
589824 \text{ Ans.}
\end{array}$$

(*b*)

$$\begin{array}{r}
5005\\
505\\
\hline
25025\\
250250\\
\hline
2527525 \text{ Ans.}
\end{array}$$

(*c*)

$$\begin{array}{r}
43911\\
8420\\
\hline
878220\\
175644\\
351288\\
\hline
369730620 \text{ Ans.}
\end{array}$$

(25) Since there are 12 months in a year, the number of days the man works is 25 × 12 = 300 days. As he works 10 hours each day, the number of hours that he works in one year is 300 × 10 = 3,000 hours. Hence, he receives for his work 3,000 × 30 = 90,000 cents, or 90,000 ÷ 100 = $900. Ans.

(26) See Art. **71.**

(27) See Art. **77.**

(28) See Art. **73.**

(29) See Art. **73.**

(30) See Art. **75.**

(31) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(32) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(33) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$. Ans.

(34) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4 we shall have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form*. Ans.

(35) In order to reduce a mixed number to an improper fraction, we must *multiply the whole number by the denominator of the fraction* and *add the numerator of the fraction to that product*. This *result* is the *numerator of the improper fraction*, of which the *denominator is the denominator of the fractional part of the mixed number*.

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$, hence in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$; $\frac{56}{8}$ plus the $\frac{7}{8}$ of the mixed number $= \frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\frac{5}{16} = \frac{(13 \times 16) + 5}{16} = \frac{213}{16};\quad 10\frac{3}{4} = \frac{(10 \times 4) + 3}{4} = \frac{43}{4}.$$

(36) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator 13 by the denominator 2. 2 is contained in 13 six times, with 1 remaining. This 1 remaining is written over the denominator 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many units (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) unit remaining. Hence, $\frac{13}{12} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number. Ans.

$\frac{17}{4} = 4\frac{1}{4}$. Ans. $\frac{69}{16} = 4\frac{5}{16}$. Ans. $\frac{16}{8} = 2$. Ans. $\frac{67}{64} = 1\frac{3}{64}$. Ans.

(37) In division of fractions, *invert the divisor* (or, in other words, turn it upside down) *and proceed as in multiplication.*

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112$. Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}$. Ans

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}$. Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1{,}808}{448} = \frac{452}{112} = \frac{113}{28}$

$$\begin{array}{l} 28)\,113\,(4\frac{1}{28}. \text{ Ans.} \\ \quad\;\; \underline{112} \\ \qquad\;\; 1 \end{array}$$

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4) + 3}{4} = \frac{60 + 3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}$.

$$\begin{array}{l} \dfrac{18}{5}\big)\,18\,\big(\,3\frac{3}{5} \quad \text{Ans.} \\ \qquad 15 \\ \qquad \overline{\ \ 3} \end{array}$$

(38) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1 + 2 + 5}{8} = \frac{8}{8} = 1.$ Ans.

When the *denominators* of the fractions to be added *are alike*, we know that the units are divided into the *same number of parts* (in this case *eighths*); we, therefore, *add the numerators* of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$ or 1 as the sum.

(39) When the *denominators* are *not alike* we know that the units are divided into *unequal parts*, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to sixteenths and then add their numerators.

$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1 \times 4}{4 \times 4} = \frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3 \times 2}{8 \times 2} = \frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16} + \frac{6}{16} + \frac{5}{16} = \frac{4 + 6 + 5}{16} = \frac{15}{16}$. Ans.

(40) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5}{8} \times \frac{2}{2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers we have $\frac{10}{16} + \frac{7}{16} = \frac{10 + 7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

$$\begin{array}{r} 42 \\ 31 \\ 9 \\ 1\frac{1}{16} \\ \hline 83\frac{1}{16} \end{array}$$ Ans.

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

(41) $29\frac{3}{4}+50\frac{5}{8}+41+69\frac{3}{16}=?$ $\frac{3}{4}=\frac{3\times 4}{4\times 4}=\frac{12}{16}$.

$\frac{5}{8}=\frac{5\times 2}{8\times 2}=\frac{10}{16}$. $\frac{12}{16}+\frac{10}{16}+\frac{3}{16}=\frac{12+10+3}{16}=\frac{25}{16}=1\frac{9}{16}$

The problem now becomes $29+50+41+69+1\frac{9}{16}=?$

29 square inches.
50 square inches.
41 square inches.
69 square inches.
$1\frac{9}{16}$ square inches.
$190\frac{9}{16}$ square inches. Ans.

(42) (*a*) $\dfrac{7}{\frac{3}{16}}=7\div\frac{3}{16}=7\times\frac{16}{3}=\frac{7\times 16}{3}=\frac{112}{3}=37\frac{1}{3}$. Ans.

The line between 7 and $\frac{3}{16}$ means that 7 is to be divided by $\frac{3}{16}$.

(*b*) $\dfrac{\frac{15}{32}}{\frac{5}{8}}=\frac{15}{32}\div\frac{5}{8}=\frac{15}{32}\times\frac{8}{5}=\dfrac{\overset{3}{\cancel{15}}\times\cancel{8}}{\underset{4}{\cancel{32}}\times\cancel{5}}=\frac{3}{4}$. Ans.

(*c*) $\dfrac{\frac{4+3}{2+6}}{5}=\dfrac{\frac{7}{8}}{5}=\frac{7}{8\times 5}=\frac{7}{40}$. (See Art. **131.**) Ans.

(43) $\frac{7}{8}$ = value of the fraction, and 28 = the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32}=\frac{7}{8}$. Hence, 32 is the required denominator. Ans.

(44) (*a*) $\frac{7}{8}-\frac{7}{16}=?$ When the *denominators* of fractions are *not alike* it is evident that the units are divided into *unequal parts*, therefore, before subtracting, *reduce the* *fractions to fractions having a common denominator.* Then, *subtract the numerators, and place the remainder over the common denominator.*

$$\frac{7\times 2}{8\times 2}=\frac{14}{16}.\quad \frac{14}{16}-\frac{7}{16}=\frac{14-7}{16}=\frac{7}{16}.\quad \text{Ans.}$$

(*b*) $13-7\frac{7}{16}=?$ This problem may be solved in two ways:

First: $13=12\frac{16}{16}$, since $\frac{16}{16}=1$, and $12\frac{16}{16}=12+\frac{16}{16}=12+1=13$.

$$\begin{array}{r} 12\frac{16}{16}\\ 7\frac{7}{16}\\ \hline 5\frac{9}{16}\end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12-7=5$ and $\frac{16}{16}-\frac{7}{16}=\frac{16-7}{16}=\frac{9}{16}$. $5+\frac{9}{16}=5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$13=\frac{13}{1}=\frac{13\times 16}{1\times 16}=\frac{208}{16}$. $7\frac{7}{16}=\frac{(7\times 16)+7}{16}=\frac{112+7}{16}=\frac{119}{16}$.

Subtracting, we have $\frac{208}{16}-\frac{119}{16}=\frac{208-119}{16}=\frac{89}{16}$ and

$$\frac{89}{16}=\begin{array}{l}16)89(5\frac{9}{16}\\ 80\\ \hline 9\\ \hline 16\end{array}$$

the same result that was obtained by the first method.

(*c*) $312\frac{9}{16}-229\frac{5}{32}=?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this we have $\frac{9}{16}=\frac{9\times 2}{16\times 2}=\frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312-229=83$ and $\frac{18}{32}-\frac{5}{32}=\frac{18-5}{32}=\frac{13}{32}$.

$$\begin{array}{r} 312\frac{18}{32}\\ 229\frac{5}{32}\\ \hline 83\frac{13}{32}\end{array}\quad 83+\frac{13}{32}=83\frac{13}{32}.\quad \textbf{Ans.}$$

(45) The man evidently traveled $85\frac{5}{12}+78\frac{9}{15}+125\frac{17}{35}$ miles.

Adding the fractions separately in this case,

$$\frac{5}{12}+\frac{9}{15}+\frac{17}{35}=\frac{5}{12}+\frac{3}{5}+\frac{17}{35}=\frac{175+252+204}{420}=\frac{631}{420}=1\frac{211}{420}.$$

Adding the whole numbers and the mixed number representing the sum of the fractions, the sum is $289\frac{211}{420}$ miles. Ans.

$$\begin{array}{r} 85 \\ 78 \\ 125 \\ 1\frac{211}{420} \\ \hline 289\frac{211}{420} \end{array}$$

To find the least common denominator, we have

$$\begin{array}{r|l} 5 & 12,\ 5,\ 35 \\ \hline 7 & 12,\ 1,\ 7 \\ \hline & 12,\ 1,\ 1, \text{ or } 5\times7\times12=420. \end{array}$$

(46)

$$\begin{array}{rll} & 573\frac{4}{5} \text{ tons.} & \frac{4}{5}=\frac{32}{40} \\ & 216\frac{5}{8} \text{ tons.} & \frac{5}{8}=\frac{25}{40} \\ \hline \textit{difference} & 357\frac{7}{40} \text{ tons. Ans.} & \frac{7}{40}=\textit{difference.} \end{array}$$

(47) Reducing $9\frac{1}{4}$ to an improper fraction, it becomes $\frac{37}{4}$. Multiplying $\frac{37}{4}$ by $\frac{3}{8}$, $\frac{37}{4}\times\frac{3}{8}=\frac{111}{32}=3\frac{15}{32}$ dollars. Ans

(48) Referring to Arts. **114** and **116,**

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 multiplied by $\frac{7}{8}$ of $\frac{5}{6}$ of 45 =

$$\frac{\cancel{2}\times\cancel{3}\times7\times19\times\cancel{11}\times7\times5\times\overset{\overset{3}{\cancel{15}}}{\cancel{45}}}{\cancel{3}\times4\times\cancel{11}\times\underset{4}{\cancel{20}}\times1\times8\times\underset{\cancel{3}}{\cancel{6}}\times1}=\frac{7\times19\times7\times5\times3}{4\times4\times8}=\frac{13{,}965}{128}=$$

$109\frac{13}{128}$. Ans.

(49) $\frac{3}{4}$ of $16=\frac{3}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{1}=12.$ $12\div\frac{2}{3}=\frac{\overset{6}{\cancel{12}}}{1}\times\frac{3}{\cancel{2}}=18.$ Ans.

(50) $211\frac{1}{4}\times1\frac{7}{8}=\frac{845}{4}\times\frac{15}{8}$, reducing the mixed numbers to improper fractions. $\frac{845}{4} \times \frac{15}{8} = \frac{12,675}{32}$ cents = amount paid for the lead. The number of pounds sold is evidently $\frac{12,675}{32} \div 2\frac{1}{2} = \frac{\overset{2,535}{\cancel{12,675}}}{\underset{16}{\cancel{32}}} \times \frac{\cancel{2}}{\cancel{5}} = \frac{2,535}{16} = 158\frac{7}{16}$ pounds. The amount remaining is $211\frac{1}{4} - 158\frac{7}{16} = \frac{845}{4} - \frac{2,535}{16} = \frac{3,380}{16} - \frac{2,535}{16} = \frac{845}{16} = 52\frac{13}{16}$ pounds. Ans.

(51)

	tenths.	hundredths.	
.	0	8	= *Eight* **hundredths.**

	tenths.	hundredths.	thousandths.	
.	1	3	1	= *One hundred thirty-one* **thousandths.**

tenths.	hundredths.	thousandths.	ten-thousandths.	
0	0	0	1	= *One* **ten-thousandth.**

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
0	0	0	0	2	7	= *Twenty-seven* **millionths.**

tenths.	hundredths.	thousandths.	ten-thousandths.	
0	1	0	8	= *One hundred eight* **ten-thousandths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
93.	0	1	0	1	= Ninety-three, and *one hundred one ten-thousandths*

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example above, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths**, so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths**, giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say, "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, *and* one hundred one ten thousandths."

(52) See Art. **139.**

(53) See Art. **153.**

(54) See Art. **160.**

(55) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of *tenths*, *hundredths*, *thousandths*, etc., of a unit, and is expressed by placing a period (.), called a decimal point, to the left of the figures of the number, and omitting the denominator.

(56) See Art. **165.**

(57) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the quotient, or .5, since only *one* cipher was annexed to the numerator. Ans.

$\frac{7}{8}$

```
8)7.000
  .875   Ans.
```

$\frac{5}{32}$

```
32)5.00000(.15625  Ans.
   32
   ---
   180
   160
   ----
    200
    192
    ---
      80
      64
      ---
      160
      160
      ---
```

Since $.65 = \frac{65}{100}$, then, $\frac{65}{100}$ must equal .65. Or, when the denominator is 10, 100, 1000, etc., point off as many places in the numerator as there are ciphers in the denominator. Doing so, $\frac{65}{100} = .65$. Ans.

$\frac{125}{1000} = .125$. Ans.

(58) (*a*) This example, written in the form of a fraction, means that the numerator (32.5 + .29 + 1.5) is to be divided by the denominator (4.7 + 9). The operation is as follows:

$$\frac{32.5 + .29 + 1.5}{4.7 + 9} = ?$$

```
          32.5
        +   .29
        +  1.5
13.7 ) 34.29000 ( 2.5029  Ans.
       274
       ---
        689
        685
        ----
          400
          274
          ----
          1260
          1233
          ----
            27
```

```
   4.7
 + 9.0
  ----
  13.7
```

Since there are 5 decimal places in the dividend and 1 in the divisor, there are 5 − 1 or 4 places to be pointed off in the quotient. The fifth figure of the decimal is evidently less than .5.

(*b*) Here again the problem is to divide the numerator, which is $(1.283 \times \overline{8+5})$, by the denominator, which is 2.63 The operation is as follows:

$$\frac{1.283 \times \overline{8+5}}{2.63} = ? \qquad \overline{8+5} = 13.$$

```
           1.283
         ×    13
          3849
         1283
  2.63)16.679000(6.3418  Ans.
       1578
         899              480
         789              263
        1100             2170
        1052             2104
          480              66
```

(*c*) $$\frac{\overline{589+27} \times \overline{163-8}}{25+39} = ?$$

```
     589
   +  27
     616

      25
     +39
      64
```

```
        163
      -   8
        155
      × 616
        930
       155
      930
   64)95480.000(1491.875
      64                Ans
      314
      256
       588
       576
        120
         64
         560
         512
          480
          448
           320
           320
```

There are three decimal places in the quotient, since three ciphers were annexed to the dividend.

(*d*) $\dfrac{\overline{40.6+7.1}\times(3.029-1.874)}{6.27+8.53-8.01}=?$

$$\begin{array}{r}40.6\\ +\ \ 7.1\\ \hline 47.7\end{array}$$

$$\begin{array}{r}6.27\\ +\ 8.53\\ \hline 14.80\\ -\ \ 8.01\\ \hline 6.79\end{array}$$

6 decimal places in the dividend − 2 decimal places in the divisor = 4 decimal places to be pointed off in the quotient.

$$\begin{array}{r}3.029\\ -\ 1.874\\ \hline 1.155\\ \times\ \ 47.7\\ \hline 8085\\ 8085\\ 4620\end{array}$$

$$\begin{array}{l}6.79\overline{)55.093500}(8.1139.\quad \text{Ans}\\ 5432\\ \overline{773}\\ 679\\ \overline{945}\\ 679\\ \overline{2660}\\ 2037\\ \overline{6230}\\ 6111\\ \overline{119}\end{array}$$

(59) $.875=\dfrac{875}{1{,}000}=\dfrac{175}{200}=\dfrac{7}{8}$ of a foot.

1 foot = 12 inches.

$\dfrac{7}{8}$ of 1 foot $=\dfrac{7}{\underset{2}{\cancel{8}}}\times\dfrac{\overset{3}{\cancel{12}}}{1}=\dfrac{21}{2}=10\dfrac{1}{2}$ inches. Ans.

(60) 12 inches = 1 foot.

$\dfrac{3}{16}$ of an inch $=\dfrac{3}{16}\div12=\dfrac{\cancel{3}}{16}\times\dfrac{1}{\underset{4}{\cancel{12}}}=\dfrac{1}{64}$ of a foot.

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, 6 − 0 = 6 places to be pointed off.

$\frac{1}{64}$) 1.000000(.015625 Ans.
64
360
320
400
384
160
128
320
320

(61) If 1 cubic inch of water weighs .03617 of a pound, the weight of 1,500 cubic inches will be .03617 × 1,500 = 54.255 lb.

.03617 lb.
1500
1808500
3617
54.25500 lb. Ans.

(62) 72.6 feet of fencing at $.50 a foot would cost

72.6 × .50, or $36.30.
.50
$36.300

If, by selling a carload of coal at a profit of $1.65 per ton, I make $36.30, then there must be as many tons of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

1.65) 36.30 (22 tons. Ans.
330
330
330

(63) $231\ )\ 17892.00000\ (\ 77.45454$, or 77.4545 to four decimal places. Ans.

$$\begin{array}{r}
1617\\
\hline
1722\\
1617\\
\hline
1050\\
924\\
\hline
1260\\
1155\\
\hline
1050\\
924\\
\hline
1260\\
1155\\
\hline
1050
\end{array}$$

(64) $$\frac{\overset{37.13}{\cancel{74.26}} \times \overset{2}{\cancel{24}} \times \overset{.0952}{\cancel{3.1416}} \times 19 \times 19 \times 350}{\underset{1,000}{\cancel{33,000}} \times \cancel{12} \times \underset{2}{\cancel{4}}} =$$

$$\frac{37.13 \times .0952 \times 19 \times 19 \times 350}{1,000} = \frac{446,618.947600}{1,000} =$$

446.619 to three decimal places. Ans.

$$\begin{array}{r}
37.13\\
.0952\\
\hline
7426\\
18565\\
33417\\
\hline
3.534776
\end{array}
\qquad
\begin{array}{r}
19\\
19\\
\hline
171\\
19\\
\hline
361
\end{array}
\qquad
\begin{array}{r}
361\\
350\\
\hline
18050\\
1083\\
\hline
126350
\end{array}
\qquad
\begin{array}{r}
3.534776\\
126350\\
\hline
176738800\\
10604328\\
21208656\\
7069552\\
3534776\\
\hline
446618.947600
\end{array}$$

(65) See Art. **174.** Applying rule in Art. **175.**

(*a*) $.7928 \times \frac{64}{64} = \frac{50.7392}{64} = \frac{51}{64}$. Ans.

(*b*) $.1416 \times \frac{32}{32} = \frac{4.5312}{32} = \frac{5}{32}$. Ans.

(*c*) $.47915 \times \frac{16}{16} = \frac{7.6664}{16} = \frac{8}{16} = \frac{1}{2}$. Ans.

(66) In subtraction of decimals, *place the decimal points directly under each other*, and proceed as in the subtraction of whole numbers, placing the *decimal point in the remainder directly under the decimal points above.*

(*a*)
$$\begin{array}{r} 709.6300 \\ .8514 \\ \hline 708.7786 \end{array}\quad \text{Ans.}$$

In the above example we proceed as follows: We can not subtract 4 ten-thousandths from 0 ten-thousandths, and, as there are no thousandths, we take 1 hundredth from the three hundredths. 1 *hundredth* = 10 *thousandths* = 100 *ten-thousandths.* 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 *thousandths* + 6 *ten-thousandths.* Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths makes it but 2 hundredths now). Since we can not do this, we take 1 tenth from 6 tenths. 1 tenth (= 10 hundredths) + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this can not be done, we take 1 unit from the 9 units. 1 *unit* = 10 *tenths;* 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (one unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

(*b*)
$$\begin{array}{r} 81.963 \\ 1.700 \\ \hline 80.263 \end{array}\quad \text{Ans.}$$

(*c*)
$$\begin{array}{r} 18.00 \\ .18 \\ \hline 17.82 \end{array}\quad \text{Ans.}$$

(*d*)
$$\begin{array}{r} 1.000 \\ .001 \\ \hline .999 \end{array}\quad \text{Ans.}$$

(*e*) $872.1 - (.8721 + .008) = ?$ In this problem we are to subtract $(.8721 + .008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

$$\begin{array}{rl} .8721 & \\ .008 & \\ \hline .8801 & \textit{sum.} \end{array}$$

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

$$\begin{array}{rl} 872.1000 & \\ .8801 & \\ \hline 871.2199 & \text{Ans.} \end{array}$$

(*f*) $(5.028 + .0073) - (6.704 - 2.38) = ?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

$$\begin{array}{rl} 5.0280 & \\ .0073 & \\ \hline 5.0353 & \textit{sum.} \end{array}$$

$$\begin{array}{rl} 6.704 & \\ 2.380 & \\ \hline 4.324 & \textit{difference.} \end{array}$$

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

The sign between the parentheses indicates that the quantities obtained by performing the above operations, are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation we obtain .7113 as the final result.

$$\begin{array}{rl} 5.0353 & \\ 4.324 & \\ \hline .7113 & \text{Ans.} \end{array}$$

(67) In subtracting a decimal from a fraction, or subtracting a fraction from a decimal, either reduce the fraction to a decimal before subtracting, or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes

$$\begin{array}{r} \frac{7}{8}\big)\underline{7.000} \\ .875 \end{array}$$

$$\begin{array}{rl} .875 & \\ .807 & \\ \hline .068 & \text{Ans.} \end{array}$$

Subtracting .807 from .875 the remainder is .068, as shown.

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction we have $.875 = \frac{875}{1,000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$. Ans.

Or, by reducing $\frac{3}{8}$ to a decimal, $\begin{array}{l} \frac{3}{8}\big)\underline{3.000} \\ \quad\;\; .375 \end{array}$ and then subtracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

$$\begin{array}{r} .875 \\ \underline{.375} \\ .500 \end{array} \quad \text{Ans.}$$

(*c*) $\left(\frac{5}{32} + .435\right) - \left(\frac{21}{100} - .07\right) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reduce $\frac{5}{32}$ to a decimal and we obtain $\frac{5}{32} = .15625$ (see example 57).

Adding .15625 and .435, $\begin{array}{rr} & .15625 \\ & \underline{.435} \\ \textit{sum} & .59125 \end{array}$ $\frac{21}{100} = .21$; subtracting, $\begin{array}{rr} & .21 \\ & \underline{.07} \\ \textit{difference} & .14 \end{array}$

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is,

$$\begin{array}{rr} & .59125 \\ & \underline{.14} \\ \textit{difference} & .45125 \end{array} \quad \text{Ans.}$$

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus, $(.53 + .274) - (.000033 + .017) = ?$

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred thousandths.	millionths.	
.0	0	0	0	3	3	
.0	1	7				
.0	1	7	0	3	3	*sum.*

tenths.	hundredths.	thousandths.	
.5	3		
.2	7	4	
.8	0	4	*sum.*

$$\begin{array}{rll} & .804 & \textit{larger sum.} \\ & \underline{.017033} & \textit{smaller sum.} \\ \textit{difference} & .786967 & \text{Ans.} \end{array}$$

(68) In addition of decimals the *decimal points must be placed directly under each other*, so that *tenths* will come *under tenths*, *hundredths under hundredths*, *thousandths under thousandths*, etc. The addition is then performed as in whole numbers, *the decimal point of the sum being placed directly under the decimal points above.*

.125
.7
.089
.4005
.9
.000027
2.214527 Ans.

(69)
927.416
8.274
372.6
62.07938
1370.36938 Ans.

(70)
tenths.
hundredths.
thousandths.
ten-thousandths.
hundred-thousandths.
millionths.
.017
.2
.000047
.217047 = Ans.

(71) (*a*) There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

.107
.013
321
107
.001391 Ans.

(*b*) There are two decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0 or two decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2 or 5 decimal places in the second product.

203
2.03
609
4060
412.09
.203
123627
824180
83.65427 Ans

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parenthesis, and then perform whatever may be required by the sign before the parenthesis.

Multiply together the numbers 2.7 and 31.85.

$$\begin{array}{r} 31.85 \\ 2.7 \\ \hline 22295 \\ 6370 \\ \hline 85.995 \end{array}$$

The parenthesis shows that .316 is to be taken from 3.16.

$$\begin{array}{r} 3.160 \\ .316 \\ \hline 2.844 \end{array}$$

The product obtained by the first operation is now multiplied by the remainder obtained by performing the operation indicated by the signs within the parenthesis.

$$\begin{array}{r} 85.995 \\ 2.844 \\ \hline 343980 \\ 343980 \\ 687960 \\ 171990 \\ \hline 244.569780 \end{array} \quad \text{Ans.}$$

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

$$\begin{array}{r} 107.8 \\ +\quad 6.541 \\ \hline 114.341 \\ -\quad 31.96 \\ \hline 82.381 \\ \times\quad 1.742 \\ \hline 164762 \\ 329524 \\ 576667 \\ 82381 \\ \hline 143.507702 \end{array} \quad \text{Ans.}$$

(72) (*a*) $\left(\frac{7}{16} - .13\right) \times \overline{.625 + \frac{5}{8}} = ?$

First perform the operation indicated by the parenthesis.

$$\frac{7}{16}=\frac{7}{16})\,\begin{array}{l} 7.0000(.4375\\ \underline{64}\\ 60\\ \underline{48}\\ 120\\ \underline{112}\\ 80\\ \underline{80} \end{array}$$

We point off four decimal places since we annexed four ciphers.

$$\begin{array}{r} .4375\\ .13\\ \hline \end{array}$$

Subtracting, we obtain $.3075$

The vinculum has the same meaning as the parenthesis;

$$\frac{5}{8}=\frac{5}{8}\,\underline{)\,5.000}\atop .625$$

hence, we perform the operation indicated by it. We point off three decimal places, since three ciphers were annexed to the 5.

Adding the terms included by the vinculum, we obtain

$$\begin{array}{r} .625\\ .625\\ \hline 1.250 \end{array}$$

The final operation is to perform the work indicated by the sign between the parenthesis and the vinculum, thus,

$$\begin{array}{r} .3075\\ 1.25\\ \hline 15375\\ 6150\\ 3075\\ \hline .384375 \end{array}\ \text{Ans.}$$

(*b*) $\left(\frac{19}{32}\times .21\right)-\left(.02\times\frac{3}{16}\right)=?$

$$.21=\frac{21}{100}.\quad \frac{19}{32}\times\frac{21}{100}=\frac{399}{3200}.\quad .02=\frac{2}{100}.\quad \frac{2}{100}\times\frac{3}{16}=\frac{6}{1600}=\frac{3}{800}.$$

$$\frac{3}{800}=\frac{3\times 4}{800\times 4}=\frac{12}{3200}.\quad \frac{399}{3200}-\frac{12}{3200}=\frac{399-12}{3200}=\frac{387}{3200}.$$

Reducing $\frac{387}{3200}$ to a decimal, we obtain

$$\begin{array}{l}
\frac{387}{3200}\,)\,387.0000000\,(\,.1209375 \quad \text{Ans.}\\
\quad 3200\\
\quad\ \ 6700\\
\quad\ \ 6400\\
\quad\quad 30000\\
\quad\quad 28800\\
\quad\quad\ \ 12000\\
\quad\quad\quad 9600\\
\quad\quad\quad 24000\\
\quad\quad\quad 22400\\
\quad\quad\quad\ \ 16000\\
\quad\quad\quad\ \ 16000
\end{array}$$

Point off seven decimal places, since seven ciphers were annexed to the dividend.

(*c*) $\left(\frac{13}{4}+.013-2.17\right)\times\overline{13\frac{1}{4}-7\frac{5}{16}}=?$

$\frac{13}{4}=\frac{13}{4}\overline{)13.00}$
$\quad 3.25$

Point off two decimal places, since two ciphers were annexed to the dividend.

$$\begin{array}{r}
3.25\\
+\ .013\\
\hline
3.263\\
-\ 2.17\\
\hline
1.093
\end{array}$$

$\frac{5}{16}$ reduced to a decimal is .3125, since

$$\begin{array}{l}
\frac{5}{16}\,)\,5.0000\,(\,.3125\\
\quad 48\\
\quad\ \ 20\\
\quad\ \ 16\\
\quad\quad 40\\
\quad\quad 32\\
\quad\quad\ \ 80\\
\quad\quad\ \ 80
\end{array}$$

Point off four decimal places, since four ciphers were annexed to the dividend.

Then, $7\frac{5}{16}=7.3125$, and $13\frac{1}{4}=13.25$, since $\frac{1}{4}=\frac{1}{4}\overline{)1.00}$
$\quad .25$

$$\begin{array}{r} 13.25 \\ -\ \underline{7.3125} \\ 5.9375 \end{array} \qquad \begin{array}{r} 5.9375 \\ \times\ \underline{1.093} \\ 178125 \\ 534375 \\ \underline{593750} \\ 6.4896875 \end{array} \text{ Ans.}$$

(73) (*a*) $.875\div\frac{1}{2}=.875\div.5\left(\text{since } \frac{1}{2}=.5\right)=1.75$. Ans. Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875=\frac{7}{8}$, since $.875=\frac{875}{1,000}=\frac{175}{200}=\frac{35}{40}=\frac{7}{8}$; then, $\frac{7}{8}\div\frac{1}{2}=\frac{7}{\underset{4}{\cancel{8}}}\times\frac{\cancel{2}}{1}=\frac{7}{4}=1\frac{3}{4}$. Since $\frac{3}{4}=$ $\begin{array}{l} 4)\,3.00\,(.75, \\ \underline{2\,8} \\ 2\,0 \\ \underline{2\,0} \end{array}$ $1\frac{3}{4}=1.75$, the same answer as above.

(*b*) $\frac{7}{8}\div.5=\frac{7}{8}\div\frac{1}{2}\left(\text{since } .5=\frac{1}{2}\right)=\frac{7}{\underset{4}{\cancel{8}}}\times\frac{\cancel{2}}{1}=\frac{7}{4}=1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8}=.875$; $.875\div.5=1.75$. Since there are three decimal places in the dividend and one in the divisor, there are 3 — 1, or 2 decimal places in the quotient.

(*c*) $\frac{.375\times\frac{1}{4}}{\frac{5}{16}-.125}=?$ We shall solve this problem by first reducing the decimals to their equivalent common fractions.

$.375=\frac{375}{1,000}=\frac{75}{200}=\frac{15}{40}=\frac{3}{8}$. $\frac{3}{8}\times\frac{1}{4}=\frac{3}{32}$, or the value of the numerator of the fraction.

$.125=\frac{125}{1,000}=\frac{25}{200}=\frac{1}{8}$. Reducing $\frac{1}{8}$ to sixteenths, we have $\frac{1\times2}{8\times2}=\frac{2}{16}$. Then, $\frac{5}{16}-\frac{2}{16}=\frac{3}{16}$, or the value of the denominator of the fraction. The problem is now reduced to

$$\frac{\frac{3}{32}}{\frac{3}{16}} = ? \quad \frac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{\cancel{3}}{\underset{2}{\cancel{32}}} \times \frac{\cancel{16}}{\cancel{3}} = \frac{1}{2} \text{ or } .5. \quad \text{Ans.}$$

(74) $\dfrac{1.25 \times 20 \times 3}{\dfrac{87 + (11 \times 8)}{459 + 32}} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{r} 1.25 \\ \times \quad 20 \\ \hline 25.00 \\ \times \quad 3 \\ \hline 75 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\dfrac{87 + (11 \times 8)}{459 + 32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The numerator is combined as though it were written $87 + (11 \times 8)$, and its result is

$$\begin{array}{r} 11 \\ 8 \text{ times} \\ \hline 88 \\ +87 \\ \hline 175 \end{array}$$

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\overset{3}{\cancel{75}} \times 491}{\underset{7}{\cancel{175}}} = \frac{1,473}{7} = 210\frac{3}{7}. \quad \text{Ans.}$$

(75) 1 plus $.001 = 1.001$. $.01$ plus $.000001 = .010001$. And $1.001 - .010001 =$

$$\begin{array}{r} 1.001 \\ .010001 \\ \hline .990999 \end{array} \quad \text{Ans.}$$

ARITHMETIC.

(SECTION 4.)

(QUESTIONS 76–117.)

(76) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. Hence, 25% of 8,428 lb. $= .25 \times 8{,}428$ lb. $= 2{,}107$ lb. Ans.

(77) Here \$100 is the base and $1\% = .01$ is the rate. Then, $.01 \times \$100 = \$1$. Ans.

(78) $\frac{1}{2}\%$ means one-half of one per cent. Since 1% is .01, $\frac{1}{2}\%$ is .005, for, $\dfrac{2\,\underline{).010}}{\;\;.005}$. And $.005 \times \$35{,}000 = \$175$. Ans.

(79) Here 50 is the base, 2 is the percentage, and it is required to find the rate. Applying rule, Art. **193,**

$$\text{rate} = \text{percentage} \div \text{base};$$
$$\text{rate} = 2 \div 50 = .04 \text{ or } 4\%. \quad \text{Ans.}$$

(80) By Art. **193,** rate = percentage ÷ base.* As percentage = 10 and base = 10, we have rate $= 10 \div 10 = 1 = 100\%$. Hence, 10 is 100% of 10. Ans.

(81) (*a*) Rate = percentage ÷ by base. Art. **193.** As percentage = \$176.54 and base = \$2,522, we have

$$\text{rate} = 176.54 \div 2{,}522 = .07 = 7\%. \quad \text{Ans.}$$

$$2522\,\underline{)\,176.54}$$
$$\qquad\qquad .07$$

---

* Remember that an expression of this form means that the first term is to be *divided by* the second term. Thus, as above, it means percentage *divided by* base.

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(*b*) Base = percentage ÷ rate. Art. **192.**

As percentage = 16.96 and rate = 8% = .08, we have

base = 16.96 ÷ .08 = 212. Ans.

$$\begin{array}{r@{\,}l} .08 & \underline{)\,16.96} \\ & \quad 212 \end{array}$$

(*c*) Amount is the sum of the base and percentage; hence the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base. Art. **193.**

Therefore, rate = 3.2025 ÷ 213.5 = .015 = $1\frac{1}{2}\%$. Ans.

$$\begin{array}{r@{\,}l} 213.5\,) & 3.2025\,(\,.015 = 1\tfrac{1}{2}\% \\ & \underline{2135} \\ & 10675 \\ & \underline{10675} \end{array}$$

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base − the difference = the percentage. Base = 207 and difference = 201.825, hence percentage = 207 − 201.825 = 5.175.

Rate = percentage ÷ base. Art. **193.**

Therefore, rate = 5.175 ÷ 207 = .025 = $.02\frac{1}{2} = 2\frac{1}{2}\%$. Ans.

$$\begin{array}{r@{\,}l} 207\,) & 5.175\,(\,.025 \\ & \underline{414} \\ & 1035 \\ & \underline{1035} \end{array}$$

(82) In this problem $5,500 is the amount, since it equals what he paid for the farm + what he gained; 15% is the rate, and the cost (to be found) is the base. Applying rule, Art. **197,**

base = amount ÷ (1 + rate); hence,
base = $5,500 ÷ (1 + .15) = $4,782.61. Ans.

$$
\begin{array}{l}
1.15)5500.0000(4782.61\\
\underline{460}\\
900\\
\underline{805}\\
950\\
\underline{920}\\
300\\
\underline{230}\\
700\\
\underline{690}\\
100\\
\underline{115}
\end{array}
$$

The example can also be solved as follows: $100\% =$ cost; if he gained 15%, then $100 + 15 = 115\% = \$5,500$, the selling price.

If $115\% = \$5,500$, $1\% = \frac{1}{115}$ of $\$5,500 = \$47.8261$, and 100%, or the cost, $= 100 \times \$47.8261 = \$4,782.61$. Ans.

(83)

$$
\begin{array}{llr}
24\ \% \text{ of } \$950 = .24\ \times 950 = & & \$228\\
12\frac{1}{2}\% \text{ of } \$950 = .125 \times 950 = & & 118.75\\
\underline{17\ \%} \text{ of } \$950 = .17\ \times 950 = & & \underline{161.50}\\
53\frac{1}{2}\% \text{ of } \$950 & = & \$508.25
\end{array}
$$

The total amount of his yearly expenses, then, is $508.25, hence his savings are $\$950 - \$508.25 = \$441.75$. Ans.

Or, as above, $24\% + 12\frac{1}{2}\% + 17\% = 53\frac{1}{2}\%$, the total percentage of expenditures; hence, $100\% - 53\frac{1}{2}\% = 46\frac{1}{2}\% =$ per cent. saved. And $\$950 \times .465 = \$441.75 =$ his yearly savings. Ans.

(84) The percentage is 961.38, and the rate is $37\frac{1}{2}$. By **Art. 192,**

$$
\begin{aligned}
\text{Base} &= \text{percentage} \div \text{rate}\\
&= 961.38 \div .375 = 2,563.68, \text{ the number. Ans.}
\end{aligned}
$$

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38 and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

$$\begin{array}{l}
.375)961.38000(2563.68 \\
\quad 750 \\
\hline
\quad 2113 \\
\quad 1875 \\
\hline
\quad\; 2388 \\
\quad\; 2250 \\
\hline
\quad\;\; 1380 \\
\quad\;\; 1125 \\
\hline
\quad\;\;\; 2550 \\
\quad\;\;\; 2250 \\
\hline
\quad\;\;\;\; 3000 \\
\quad\;\;\;\; 3000 \\
\hline
\end{array}$$

(85) Here $1,125 is 30% of some number; hence, $1,125 = the percentage, 30% = the rate, and the required number is the base. Applying rule, Art. **192,**

Base = percentage ÷ rate = $1,125 ÷ .30 = $3,750.

Since $3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$ of $3,750 = $1,250, and $\frac{4}{4}$ or the entire property, is 4 × $1,250 = $5,000. Ans.

(86) Here $4,810 is the difference and 35% the rate. By Art. **198,**

Base = difference ÷ (1 − rate)
= $4,810 ÷ (1 − .35) = $4,810 ÷ .65 = $7,400. Ans.

$$\begin{array}{l}
.65)4810.00(7400 \\
\quad 455 \\
\hline
\quad\; 260 \\
\quad\; 260 \\
\hline
\quad\quad 00
\end{array}
\qquad\qquad
\begin{array}{r}
1.00 \\
.35 \\
\hline
.65
\end{array}$$

Solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is $4,810.

If 65% = \$4,810, 1% = $\frac{1}{65}$ of 4,810 = \$74, and 100% = 100 × \$74 = \$7,400. Ans.

(87) In this example the sales on Monday amounted to \$197.55, which was $12\frac{1}{2}$% of the sales for the entire week; i. e., we have given the percentage, \$197.55, and the rate, $12\frac{1}{2}$%, and the required number (or the amount of sales for the week) equals the base. By Art. **192**,

Base = percentage ÷ rate = \$197.55 ÷ .125;

or,

$$\begin{array}{l}
.125)197.5500(1580.4 \quad \text{Ans.} \\
\underline{125} \\
725 \\
\underline{625} \\
1005 \\
\underline{1000} \\
500 \\
\underline{500}
\end{array}$$

Therefore, base = \$1,580.40, which also equals the sales for the week.

(88) 16.5 miles = $12\frac{1}{2}$% of the entire length of the road. We wish to find the *entire* length.

16.5 miles is the percentage, $12\frac{1}{2}$% is the rate, and the entire length will be the base. By Art. **192**,

Base = percentage ÷ rate = 16.5 ÷ .$12\frac{1}{2}$.

$$\begin{array}{l}
.125)16.500(132 \text{ miles.} \quad \text{Ans.} \\
\underline{125} \\
400 \\
\underline{375} \\
250 \\
\underline{250}
\end{array}$$

(89) Here we have given the difference, or \$35, and the rate, or 60%, to find the base. We use the rule in Art. **198,**

$$\begin{aligned}\text{Base} &= \text{difference} \div (1 - \text{rate})\\ &= \$35 \div (1 - .60) = \$35 \div .40 = \$87.50. \quad \text{Ans.}\end{aligned}$$

.40)35.000(87.5
320
300
280
200
200

Or, 100% = whole debt; 100% − 60% = 40% = \$35.

If 40% = \$35, then $1\% = \frac{1}{40}$ of $\$35 = \frac{35}{40}$, and $100\% = \frac{35}{40} \times 100 = \$87.50$. Ans.

(90) 28 rd. 4 yd. 2 ft. 10 in. to inches.

× 5½
154
+ 4
158 yards
× 3
474
+ 2
476 feet
× 12
5712
+ 10
5722 inches. Ans.

Since there are 5½ yards in one rod, in 28 rods there are 28 × 5½ or 154 yards; 154 yards plus 4 yards = 158 yards. There are 3 feet in one yard; therefore, in 158 yards there are 3 × 158 or 474 feet; 474 feet + 2 feet = 476 feet. There are 12 inches in one foot, and in 476 feet there are 12 × 476 or 5,712 inches; 5,712 inches + 10 inches = 5,722 inches. Ans.

(91)
12) 5722 inches.
3) 476 + 10 inches.
5½) 158 + 2 feet.
28 + 4 yards.

Ans. = 28 rd. 4 yd. 2 ft. 10 in.

Explanation.—There are 12 inches in 1 foot; hence, in 5,722 inches there are as many feet as 12 is contained times in 5,722 inches, or 476 ft. and 10 inches remaining. Write these 10 inches as a remainder. There are 3 feet in 1 yard; hence, in 476 feet there are as many yards as 3 is contained times in 476 feet, or 158 yards and 2 feet remaining. There are $5\frac{1}{2}$ yards in one rod; hence, in 158 yards there are 28 rods and 4 yards remaining. Then, in 5,722 inches there are 28 rd. 4 yd. 2 ft. 10 in.

(92)

$$\begin{array}{rl}
5 & \text{weeks } 3.5 \text{ days.} \\
\times\ \underline{7} & \\
35 & \text{days in 5 weeks.} \\
+\ \underline{3.5} & \\
38.5 & \text{days.}
\end{array}$$

Then, we find how many seconds there are in 38.5 days.

$$\begin{array}{rl}
38.5 & \text{days} \\
\times\ \underline{24} & \text{hours in one day.} \\
1540 & \\
\underline{770} & \\
924.0 & \text{hours in 38.5 days.} \\
\times\ \underline{60} & \text{minutes in one hour.} \\
55440 & \text{minutes in 38.5 days.} \\
\times\ \underline{60} & \text{seconds in one minute.} \\
3326400 & \text{seconds in 38.5 days.} \quad \text{Ans.}
\end{array}$$

(93) Since there are 24 gr. in 1 pwt., in 13,750 gr. there are as many pennyweights as 24 is contained times in 13,750, or 572 pwt. and 22 gr. remaining. Since there are 20 pwt. in 1 oz., in 572 pwt. there are as many ounces as 20 is contained times in 572, or 28 oz. and 12 pwt. remaining.

Since there are 12 oz. in 1 lb. (Troy), in 28 oz. there are as many pounds as 12 is contained times in 28, or 2 lb. and 4 oz. remaining. We now have the pounds and ounces required by the problem; therefore, in 13,750 gr. there are 2 lb. 4 oz. 12 pwt. 22 gr.

$$\begin{array}{l} 24\,\underline{)\,13750}\text{ gr.} \\ \quad 20\,\underline{)\,572}\text{ pwt.} + 22\text{ gr.} \\ \qquad 12\,\underline{)\,28}\text{ oz.} + 12\text{ pwt.} \\ \qquad\qquad 2\text{ lb.} + 4\text{ oz.} \end{array}$$

Ans. = 2 lb. 4 oz. 12 pwt. 22 gr.

(94)

$$\begin{array}{l} 100\,\underline{)\,4763254}\text{ li.} \\ \quad 80\,\underline{)\,47632} + 54\text{ li.} \\ \qquad\qquad 595 + 32\text{ ch.} \end{array}$$

Ans. = 595 mi. 32 ch. 54 li.

Explanation.—There are 100 li. in one chain; hence, in 4,763,254 li. there are as many chains as 100 is contained times in 4,763,254 li., or 47,632 ch. and 54 li. remaining. Write the 54 li. as a remainder. There are 80 ch. in one mile; hence, in 47,632 ch. there are as many miles as 80 is contained times in 47,632 ch., or 595 miles and 32 ch. remaining. Then, in 4,763,254 li. there are 595 mi. 32 ch. 54 li.

(95)

$$\begin{array}{l} 1728\,\underline{)\,764325}\text{ cu. in.} \\ \qquad 27\,\underline{)\,442} + 549\text{ cu. in.} \\ \qquad\qquad 16\text{ cu. yd.} + 10\text{ cu. ft.} \end{array}$$

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

Explanation.—There are 1,728 cu. in. in one cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cu. ft. and 549 cu. in. remaining. Write the 549 cu. in. as a remainder. There are 27 cu. ft. in one cubic yard; hence, in 442 cu. ft. there are as many cubic yards as 27 is contained times in 442 cu. ft., or 16 cu. yd. and 10 cu. ft. remaining. Then, in 764,325 cu. in. there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(96) We must arrange the different terms in columns, taking care to have like denominations in the same column.

	rd.	yd.	ft.	in.	
	2	2	2	3	
		4	1	9	
			2	7	
	3	$2\frac{1}{2}$	0	7	
or	3	2	2	1	Ans.

Explanation.—We begin to add at the right-hand column. $7 + 9 + 3 = 19$ in.; as 12 in. make one foot, 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the next column.

1 (reserved) + 2 + 1 + 2 = 6 ft. Since 3 ft. make 1 yard, 6 ft. = 2 yd. and 0 ft. remaining. Place the cipher in the column of feet and reserve the 2 yd. for the next column.

2 (reserved) + 4 + 2 = 8 yd. Since $5\frac{1}{2}$ yd. = 1 rod, 8 yd. = 1 rd. and $2\frac{1}{2}$ yd. Place $2\frac{1}{2}$ yd. in the yards column, and reserve 1 rd. for the next column; 1 (reserved) + 2 = 3 rd.

Ans. =	3 rd.	$2\frac{1}{2}$ yd.	0 ft.	7 in.	
or,	3 rd.	2 yd.	1 ft.	13 in.	
or,	3 rd.	2 yd.	2 ft.	1 in.	Ans.

(97) We write the compound numbers so that the units of the same denomination shall stand in the same column. Beginning to add with the lowest denomination, we find that the sum of the gills is 1 + 2 + 3 = 6. Since there are 4 gi. in 1 pint, in 6 gi. there are as many pints as 4 is contained times in 6, or 1 pt. and 2 gi. We place 2 gi. under the gills column and reserve the 1 pt. for the pints column; the sum of the pints is 1 (reserved) + 5 + 1 + 1 = 8. Since there are 2 pt. in 1 quart, in 8 pt. there are as many quarts as 2 is contained times in 8, or 4 qt. and 0 pt. We place the cipher under the column of pints and reserve the 4 for the quarts column. The sum of the quarts is 4 (reserved) + 8 + 3 = 15. Since there are 4 qt. in 1 gallon, in 15 qt. there are as many gallons as 4 is contained times in 15, or 3 gal. and 3 qt. remaining. We now place the 3 under the quarts column and reserve the 3 gal. for the gallons column. The sum of the gallons column is 3 (reserved) + 4 + 6 + 3 = 16 gal. Since we can not reduce 16 gal. to any higher denomination, we have 16 gal. 3 qt. 0 pt. and 2 gi. for the answer.

gal.	qt.	pt.	gi.
3	3	1	3
6	0	1	2
4	0	0	1
	8	5	0
16 gal.	3 qt.	0 pt.	2 gi.

(98) Reduce the grains, pennyweights, and ounces to higher denominations.

24) 240 gr.	20) 125 pwt.	12) 50 oz.
10 pwt.	6 oz. 5 pwt.	4 lb. 2 oz.

Then, 3 lb. + 4 lb. 2 oz. + 6 oz. 5 pwt. + 10 pwt. =

lb.	oz.	pwt.	
3			
4	2		
	6	5	
		10	
7 lb.	8 oz.	15 pwt.	Ans.

(99) Since "seconds" is the lowest denomination in this problem, we find their sum first, which is 11 + 29 + 25 + 30 + 12, or 107 seconds. Since there are 60 seconds in 1 minute, in 107″ there are as many minutes as 60 is contained times in 107, or 1 minute and 47 seconds remaining. We place the 47 under the seconds column and reserve the 1 for the minutes column. The sum of the minutes is 1 (reserved) + 17 + 26 + 19 + 16, or 79. Since there are 60 minutes in 1 degree, in 79 minutes there are as many degrees as 60 is contained times in 79, or 1 degree and 19 minutes remaining. We place the 19 under the minutes column and reserve the 1 degree for the degrees column. The sum of the degrees is 1 (reserved) + 10 + 20 + 13 + 11, or 55 degrees. Since we can not reduce 55 degrees to any higher denominations, we have 55° 19′ 47″ for the answer.

deg.	min.	sec.
11	16	12
13	19	30
20	0	25
0	26	29
10	17	11
55°	19′	47″

(100) Since "inches" is the lowest denomination in this problem, we find their sum first, which is 11 + 8 + 6, or 25 inches. Since there are 12 inches in 1 foot, in 25 inches there are as many feet as 12 is contained times in 25, or 2 feet and 1 inch remaining. Place the 1 inch under the inches column, and reserve the 2 feet to add to the column of feet. The sum of the feet is 2 feet (reserved) + 2 + 1 = 5 feet. Since there are 3 feet in 1 yard, in 5 feet there are as many yards as 3 is contained times in 5 feet, or 1 yard and 2 feet remaining. Place the 2 feet under the column of feet, and reserve the 1 yard to add to the column of yards. The sum of the yards is 1 yard (reserved) + 4 + 5 = 10 yards. Since there are $5\frac{1}{2}$ yards in 1 rod, in 10 yards there are as many rods as $5\frac{1}{2}$ is contained times in 10, or 1 rod and $4\frac{1}{2}$ yards remaining. Place the $4\frac{1}{2}$ yards under the column of yards, and reserve the 1 rod for the column of rods. The sum of the rods is 1 (reserved) + 304 + 215 + 130 = 650 rods. Place 650 rods under the column of rods. Therefore, the sum is 650 rd. $4\frac{1}{2}$ yd. 2 ft. 1 in. Or, since $\frac{1}{2}$ a yard = 1 ft. 6 in., and since there are 320 rods in 1 mile, the sum may be expressed as 2 mi. 10 rd. 5 yd. 0 ft. 7 in. Ans.

	rd.	yd.	ft.	in.	
	130	5	1	6	
	215	0	2	8	
	304	4	0	11	
	650	$4\frac{1}{2}$	2	1	
mi.					
or, 2	10	5	0	7	Ans.

(101) Since "square links" is the lowest denomination in this problem, we find their sum first, which is 21 + 23 + 16 + 18 + 23 + 21, or 122 square links. Place 122 square links under the column of square links. The sum of the square rods is 2 + 3 + 2 + 2 + 2 + 3, or 14 square rods. Place 14 square rods under the column of square rods. The sum of the square chains is 323 square chains. Since there are 10 square chains in 1 acre, in 323 square chains there are as many acres as 10 is

A.	sq. ch.	sq. rd.	sq. li.
21	67	3	21
28	78	2	23
47	6	2	18
56	59	2	16
25	38	3	23
46	75	2	21
255	3	14	122

contained times in 323 square chains, or 32 acres and 3 square chains remaining. Place 3 square chains under the column of square chains, and reserve the 32 acres to add to the column of acres. The sum of the acres is 32 acres (reserved) + 46 + 25 + 56 + 47 + 28 + 21, or 255 acres. Place 255 acres under the column of acres. Therefore, the sum is 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li. Ans.

(102) Before we can subtract 300 ft. from 20 rd. 2 yd. 2 ft. and 9 in., we must reduce the 300 ft. to higher denominations.

Since there are 3 feet in 1 yard, in 300 feet there are as many yards as 3 is contained times in 300, or 100 yards. There are $5\frac{1}{2}$ yards in 1 rod, hence in 100 yards there are as many rods as $5\frac{1}{2}$ or $\frac{11}{2}$ is contained times in $100 = 18\frac{2}{11}$ rods.

$$100 \div \frac{11}{2} = 100 \times \frac{2}{11} = \frac{100 \times 2}{11} = \frac{200}{11}\begin{array}{l})\ 200\ (\ 18\frac{2}{11}\text{ rd.} \\ \quad\ \underline{11} \\ \quad\ \ 90 \\ \quad\ \ \underline{88} \\ \quad\ \ \ \ 2 \end{array}$$

Since there are $5\frac{1}{2}$ or $\frac{11}{2}$ yards in 1 rod, in $\frac{2}{11}$ rods there are $\frac{\cancel{2}}{\cancel{11}} \times \frac{\cancel{11}}{\cancel{2}}$, or one yard, so we find that 300 feet equals 18 rods and 1 yard. The problem now is as follows: From 20 rd. 2 yd. 2 ft. and 9 in. take 18 rd. and 1 yd.

We place the smaller number under the larger one, so that units of the same denomination fall in the same column. Beginning with the lowest denomination, we see that 0 inches from 9 inches leaves 9 inches. Going to the next higher denomination, we see that 0 feet from 2 feet leaves 2 feet. Subtracting 1 yard from 2

rd.	yd.	ft.	in.
20	2	2	9
18	1	0	0
2	1	2	9

yards, we have 1 yard remaining, and 18 rods from 20 rods leaves 2 rods. Therefore, the difference is 2 rd. 1 yd. 2 ft. 9 in. Ans.

(103)

A.	sq. rd.	sq. yd.	
114	80	25	
75	70	30	
39	9	$25\frac{1}{4}$	Ans.

Explanation.—Place the subtrahend under the minuend so that like denominations are under each other. Then begin at the right with the lowest denomination. We can not subtract 30 from 25, so we take one square rod $(= 30\frac{1}{4}$ square yards) from 80 square rods, leaving 79 square rods; adding $30\frac{1}{4}$ square yards to 25 square yards, we have $55\frac{1}{4}$ square yards; subtracting 30 from $55\frac{1}{4}$ square yards leaves $25\frac{1}{4}$ square yards; we now subtract 70 square rods from 79 square rods, which leaves 9 square rods; next, we subtract 75 acres from 114 acres, which leaves 39 acres, which we place under the column of acres.

(104) If 10 gal. 2 qt. and 1 pt. of molasses are sold from a hogshead at one time, and 26 gal. 3 qt. are sold at another time, then the total amount of molasses sold equals 10 gal. 2 qt. 1 pt. plus 26 gal. 3 qt.

gal.	qt.	pt.
10	2	1
26	3	0
37 gal.	1 qt.	1 pt.

Since the pint is the lowest denomination, we add the pints first, which equal 0 + 1, or 1 pint. We can not reduce 1 pint to any higher denomination, so we place it under the pint column. The number of quarts is 3 + 2, or 5. Since there are 4 quarts in 1 gallon, in 5 quarts there are as many gallons as 4 is contained times in 5, or 1 gallon and 1 quart remaining. We place the 1 quart under the quart column, and reserve the 1 gallon to add to the column of gallons. The number of gallons equals 1 (reserved) + 26 + 10, or 37 gallons.

If 37 gal. 1 qt. and 1 pt. are sold from a hogshead of molasses (63 gal.), there remains the difference between 63 gal. and 37 gal. 1 qt. 1 pt., or 25 gal. 2 qt. and 1 pt.

63 gal. is the same as 62 gal. 3 qt. 2 pt., since 1 gal. equals 4 qt. and 1 qt. = 2 pt.

Beginning with the lowest denomination, 1 pt. from the 2 pt. 1 pint from 2 pints leaves 1 pint. One quart from 3 quarts leaves 2 quarts, and 37 gallons from 62 gallons leaves 25 gallons. Therefore, there are 25 gal. 2 qt. and 1 pt. of molasses remaining in the hogshead. Ans.

gal.	qt.	pt.
62	3	2
37	1	1
25	2	1

(105) If a person were born June 19, 1850, in order to find how old he would be on Aug. 3, 1892, subtract the earlier date from the later date.

On August 3, 7 mo. and 3 da. have elapsed from the beginning of the year, and on June 19, 5 mo. and 19 da.

Beginning with the lowest denomination, we find that 19 days can not be taken from 3 days, so we take 1 month from 7 months. The 1 month which we took equals 30 days, for in all cases 30 days are allowed to a month. Adding 30 days to the 3 days, we have 33 days; subtracting 19 days from 33 days, we have 14 days remaining. Since we borrowed 1 month from the months column, we have 7 — 1, or 6 months remaining; subtracting 5 months from 6 months, we have 1 month remaining. 1850 from 1892 leaves 42 years. Therefore, he would be 42 years 1 month and 14 days old. Ans.

yr.	mo.	da.
1892	7	3
1850	5	19
42	1	14

(106) If a note given Aug. 5, 1890, were paid June 3, 1892, in order to find the length of time it was due, subtract the earlier date from the later date.

Beginning with the lowest denomination, we find that 5 can not be subtracted from 3, so we take a unit from the next

yr.	mo.	da.
1892	5	3
1890	7	5
1	9	28

higher denomination, which is months. The 1 month which we take equals 30 days. Adding the 30 days to the 3 days, we have 33 days. 5 days from 33 days leaves 28 days. Since we took 1 month from the months column, only 4 months remain. 7 months cannot be taken from 4 months, so we take 1 year from the years column, which equals 12 months. 12 months + 4 months = 16 months. 7 months from 16 months = 9 months. Since we took 1 year from the years column, we have 1892 — 1, or 1891 remaining. 1890 from 1891 leaves 1 year. Hence, the note ran 1 year 9 months and 28 days. Ans.

(107) Write the number of the year, month, day, hour, and minute of the earlier date under the year, month, day, hour, and minute of the later date, and subtract.

22 minutes before 8 o'clock is the same as 38 minutes after 7 o'clock. 7 o'clock P. M. is 19 hours from the beginning of the day, as there are 12 hours in the morning and 7 in the afternoon. December is 11 months from the beginning of the year.

10 o'clock A. M. is 10 hours from the beginning of the day. July is 6 months from the beginning of the year. The minuend would be the later date, or 1,888 years, 11 months, 11 days, 19 hours, and 38 minutes.

The subtrahend would be the earlier date, or 1,883 years, 6 months, 3 days, 10 hours, and 16 minutes.

Subtracting, we have

yr.	mo.	da.	hr.	min.
1888	11	11	19	38
1883	6	3	10	16
5	5	8	9	22

or, 5 yr. 5 mo. 8 da. 9 hr. and 22 min. Ans.

16 minutes subtracted from 38 minutes leaves 22 minutes; 10 hours from 19 hours leaves 9 hours; 3 days from 11 days leaves 8 days; 6 months subtracted from 11 months leaves 5 months; 1,883 from 1,888 leaves 5 years.

(108) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

17 ft.	3 in.
	51
879 ft.	9 in.

and begin at the right to multiply. 51 × 3 = 153 in. As there are 12 inches in 1 foot, in 153 in. there are as many feet as 12 is contained times in 153, or 12 feet and 9 inches remaining. Place the 9 inches under the inches, and reserve the 12 feet. 51 × 17 ft. = 867 ft. 867 ft. + 12 ft. (reserved) = 879 ft.

879 feet can be reduced to higher denominations by dividing by 3 feet to find the number of yards, and by $5\frac{1}{2}$ yards to find the number of rods.

3) 8 7 9 ft. 9 in.
5.5) 2 9 3 yd.
5 3 rd. 1½ yd.

Then, answer = 53 rd. 1½ yd. 0 ft. 9 in.; or 53 rd. 1 yd. 2 ft. 3 in.

(109)	qt.	pt.	gi.	
	3	1	3	
			4.7	
	1 8.2 qt.	0	.1	
or,	1 8 qt.	0 pt.	1.7 gi.	
or, 4 gal.	2 qt.	0 pt.	1.7 gi.	Ans.

Place the multiplier under the lowest denomination of the multiplicand, and proceed to multiply. 4.7 × 3 gi. = 14.1 gi. As 4 gi. = 1 pt., there are as many pints in 14.1 gi. as 4 is contained times in 14.1 = 3.5 pt. and .1 gi. over. Place .1 under gills and carry the 3.5 pt. forward. 4.7 × 1 pt. = 4.7 pt.; 4.7 + 3.5 pt. = 8.2 pt. As 2 pt. = 1 qt., there are as many quarts in 8.2 pt. as 2 is contained times in 8.2 = 4.1 qt. and no pints over. Place a cipher under the pints, and carry the 4.1 qt. to the next product. 4.7 × 3 qt. = 14.1; 14.1 + 4.1 = 18.2 qt. The answer now is 18.2 qt. 0 pt. .1

gi. Reducing the fractional part of a quart, we have 18 qt. 0 pt. 1.7 gi. (.2 qt. = .2 × 8 = 1.6 gi.; 1.6 + .1 gi. = 1.7 gi.). Then, we can reduce 18 qt. to gallons (18 ÷ 4 = 4 gal. and 2 qt.) = 4 gal. 2 qt. 1.7 gi. Ans.

The answer may be obtained in another and much easier way by reducing all to gills, multiplying by 4.7, and then changing back to quarts and pints. Thus,

$$\begin{array}{rl} & 3 \text{ qt.} \\ \times & 2 \text{ pt.} \\ \hline & 6 \text{ pt.} \\ + & 1 \text{ pt.} \\ \hline & 7 \text{ pt.} \\ \times & 4 \text{ gi.} \\ \hline & 28 \text{ gi.} \\ + & 3 \text{ gi.} \\ \hline & 31 \text{ gi.} \end{array}$$

3 qt. 1 pt. 3 gi. = 31 gi.
31 gi. × 4.7 = 145.7 gi.

$$\begin{array}{r@{\,}l} 4\overline{)145.7} & \text{gi.} \\ 2\overline{)36} & \text{pt.} + 1.7 \text{ gi.} \\ 18 & \text{qt.} + 0 \text{ pt.} \end{array}$$

Ans. = 18 qt. 1.7 gi.;
or, 4 gal. 2 qt. 1.7 gi.

(110) (3 lb. 10 oz. 13 pwt. 12 gr.) × 1.5 = ?

$$\begin{array}{rl} & 3 \text{ lb. } 10 \text{ oz. } 13 \text{ pwt. } 12 \text{ gr.} \\ \times & 12 \\ \hline & 36 \text{ oz.} \\ + & 10 \\ \hline & 46 \text{ oz.} \\ \times & 20 \\ \hline & 920 \text{ pwt.} \\ + & 13 \\ \hline & 933 \text{ pwt.} \\ \times & 24 \\ \hline & 22392 \text{ gr.} \\ + & 12 \\ \hline & 22404 \text{ gr.} \end{array}$$

22,404 gr. × 1.5 = 33,606 gr.

$$\begin{array}{r@{\,}l} 24\overline{)33606} & \text{gr.} \\ 20\overline{)1400} & \text{pwt.} + 6 \text{ gr.} \\ 12\overline{)70} & \text{oz.} + 0 \text{ pwt.} \\ 5 & \text{lb.} + 10 \text{ oz.} \end{array}$$

Since there are 24 gr. in 1 pwt., in 33,606 gr. there are as many pwt. as 24 is contained times in 33,606, or 1,400 pwt. and 6 gr. remaining. This gives us the number of grains in the answer. We now reduce 1,400 pwt. to higher denominations. Since there are 20 pwt. in 1 oz., in 1,400 pwt. there are as many ounces as 20 is contained times in 1,400, or 70 oz. and 0 pwt. remaining; therefore, there are 0 pwt. in the answer. We reduce 70 oz. to higher denominations. Since there are 12 oz. in 1 lb., in 70 oz. there are as many pounds as 12 is contained times in 70, or 5 lb. and 10 oz. remaining. We can not reduce 5 lb. to any higher denominations. Therefore, our answer is 5 lb. 10 oz. 6 gr.

Another but more complicated way of working this problem is as follows:

	lb.	oz.	pwt.	gr.	
	3	10	13	12	
				1.5	
	4.5	15	19.5	18	
or,	4	21	19	30	
or,	5	10	0	6	Ans.

To get rid of the decimal in the pounds, reduce .5 of a pound to ounces. Since 1 lb. = 12 oz., .5 of a pound equals .5 lb. × 12 = 6 oz. 6 oz. + 15 oz. = 21 oz. We now have 4 lb. 21 oz. 19.5 pwt. and 18 gr., but we still have a decimal in the column of pwt., so we reduce .5 pwt. to grains to get rid of it. Since 1 pwt. = 24 gr., .5 pwt. = .5 pwt. × 24 = 12 gr. 12 gr. + 18 gr. = 30 gr. We now have 4 lb. 21 oz. 19 pwt. and 30 gr. Since there are 24 gr. in 1 pwt., in 30 gr. there is 1 pwt. and 6 gr. remaining. Place 6 gr. under the column of grains and add 1 pwt. to the pwt. column. Adding 1 pwt., we have 19 + 1 = 20 pwt. Since there are 20 pwt. in 1 oz., we have 1 oz. and 0 pwt. remaining. Write the 0 pwt. under the pwt. column, and reserve the 1 oz. to the oz. column. 21 oz. + 1 oz. = 22 oz. Since there are 12 oz. in 1 lb., in 22 oz. there is 1 lb. and 10 oz. remaining. Write the 10 oz. under the ounce column, and reserve the 1 lb. to add to the lb. column. 4 lb. + 1 lb. (reserved) = 5 lb. Hence, the answer equals 5 lb. 10 oz. 6 gr.

(111) If each barrel of apples contains 2 bu. 3 pk. and 6 qt., then 9 bbl. will contain 9 × (2 bu. 3 pk. 6 qt.).

We write the multiplier under the lowest denomination of the multiplicand, which is quarts in this problem. 9 times 6 qt. equals 54 qt. There are 8 qt. in 1 pk., and in 54 qt. there are as many pecks as 8 is contained times in 54, or 6 pk. and 6 qt. We write the 6 qt. under the column of quarts, and reserve the 6 pk. to add to the product of the pecks. 9 times 3 pk. equals 27 pk.; 27 pk. plus the 6 pk. reserved equals 33 pk. Since there are 4 pk. in 1 bu., in 33 pk. there are as many bushels as 4 is contained times in 33, or 8 bu. and 1 pk. remaining. We write the 1 pk. under the column of pecks, and reserve the 8 bu. for the product of the bushels. 9 times 2 bu. plus the 8 bu. reserved equals 26 bu. Therefore, we find that 9 bbl. contain 26 bu. 1 pk. 6 qt. of apples. Ans.

$$\begin{array}{lrrr} & \text{bu.} & \text{pk.} & \text{qt.}\\ & 2 & 3 & 6\\ & & & 9\\ \cline{2-4} & 18 & 27 & 54\\ \text{or,} & 26 & 1 & 6\end{array}$$

(112) (7 T. 15 cwt. 10.5 lb.) × 1.7 = ? When the multiplier is a decimal, instead of multiplying the denominate numbers as in the case when the multiplier is a whole number, it is much easier to reduce the denominate numbers to the lowest denomination given; then, multiply that result by the decimal, and, lastly, reduce the product to higher denominations. Although the correct answer can be obtained by working examples involving decimals in the manner as in the last example, it is much more complicated than this method.

$$\begin{array}{rrl} & 7 & \text{T. 15 cwt. 10.5 lb.}\\ \times & 20 & \\ \cline{2-2} & 140 & \text{cwt.}\\ & 15 & \\ \cline{2-2} & 155 & \text{cwt.}\\ \times & 100 & \\ \cline{2-2} & 15500 & \text{lb.}\\ & 10.5 & \\ \cline{2-2} & 15510.5 & \text{lb.}\end{array}$$

15,510.5 lb. × 1.7 = 26,367.85 lb.

There are 100 lb. in 1 cwt., and in 26,367.85 lb. there are as many cwt. as 100 is contained times in 26,367.85, which equals 263 cwt. and 67.85 lb. remaining. Since we have the number of pounds for our answer, we reduce 263 cwt. to higher denominations

$$\begin{array}{r|l} 100 & 26367.85 \text{ lb.} \\ \hline 20 & 263 \text{ cwt.} + 67.85 \text{ lb.} \\ \hline & 13 \text{ T.} + 3 \text{ cwt.} \end{array}$$

There are 20 cwt. in 1 ton, and in 263 cwt. there are as many tons as 20 is contained times in 263, or 13 tons and 3 cwt. remaining. Since we cannot reduce 13 tons any higher, our answer is 13 T. 3 cwt. 67.85 lb. Or, since .85 lb. = .85 lb. × 16 = 13.6 oz., the answer may be written 13 T. 3 cwt. 67 lb. 13.6 oz.

(113)

$$\begin{array}{r|llll} 7 & 358 \text{ A.} & 57 \text{ sq. rd.} & 6 \text{ sq. yd.} & 2 \text{ sq. ft.} \\ \hline & 51 \text{ A.} & 31 \text{ sq. rd.} & 0 \text{ sq. yd.} & 8 \text{ sq. ft.} \quad \text{Ans.} \end{array}$$

We begin with the highest denomination, and divide each term in succession by 7.

7 is contained in 358 A. 51 times and 1 A. remaining. We write the 51 A. under the 358 A. and reduce the remaining 1 A. to square rods = 160 sq. rd.; 160 sq. rd. + the 57 sq. rd. in the dividend = 217 sq. rd. 7 is contained in 217 sq. rd. 31 times and 0 sq. rd. remaining. 7 is not contained in 6 sq. yd., so we write 0 under the sq. yd. and reduce 6 sq. yd. to square feet. 9 sq. ft. × 6 = 54 sq. ft. 54 sq. ft. + 2 sq. ft. in the dividend = 56 sq. ft. 7 is contained in 56 sq. ft. 8 times. We write 8 under the 2 sq. ft. in the dividend.

(114)

$$\begin{array}{r|llll} 12 & 282 \text{ bu.} & 3 \text{ pk.} & 1 \text{ qt.} & 1 \text{ pt.} \\ \hline & 23 \text{ bu.} & 2 \text{ pk.} & 2 \text{ qt.} & \frac{1}{4} \text{ pt.} \quad \text{Ans.} \end{array}$$

12 is contained in 282 bu. 23 times and 6 bu. remaining. We write 23 bu. under the 282 bu. in the dividend, and reduce the remaining 6 bu. to pecks = 24 pk. + the 3 pk. in the dividend = 27 pk. 12 is contained in 27 pk. 2 times and 3 pk. remaining. We write 2 pk. under the 3 pk. in the dividend, and reduce the remaining 3 pk. to quarts. 3 pk. = 24 qt.; 24 qt. + the 1 qt. in the dividend = 25 qt. 12 is contained in 25 qt. 2 times and 1 qt. remaining. We write

2 qt. under the 1 qt. in the dividend, and reduce 1 qt. to pints = 2 pt. + the 1 pt. in the dividend = 3 pt. 3 ÷ 12 = $\frac{3}{12}$ or $\frac{1}{4}$ pt.

(115) We must first reduce 23 miles to feet before we can divide by 30 feet. 1 mi. contains 5,280 ft.; hence, 23 mi. contain 5,280 × 23 = 121,440 ft.

121,440 ft. ÷ 30 ft. = 4,048 rails for 1 side of the track.

The number of rails for 2 sides of the track = 2 × 4,048, or 8,096 rails. Ans.

(116) In this case where both dividend and divisor are compound, reduce each to the lowest denomination mentioned in either and then divide as in simple numbers.

1 bu. 1 pk. 7 qt.
× 4
4 pk.
+ 1
5 pk.
× 8
40 qt.
+ 7
47 qt.

356 bu. 3 pk. 5 qt.
× 4
1424 pk.
+ 3
1427 pk.
× 8
11416 qt.
+ 5
11421 qt.

47) 11421 (243
94
202
188
141
141

11,421 qt. ÷ 47 qt. = 243 boxes
Ans

(117) We must first reduce 16 square miles to acres. In 1 sq. mi. there are 640 A., and in 16 sq. mi. there are 16 × 640 A. = 10,240 A.

62) 10240 A.
165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80+ sq. in. Ans.

62 is contained in 10,240 A. 165 times and 10 A. remaining. We write 165 A. under the 10,240 A. in the dividend and reduce 10 A. to sq. rd. In 1 A. there are 160 sq. rd., and in 10 A. there are $10 \times 160 = 1,600$ sq. rd. 62 is contained in 1,600 sq. rd. 25 times and 50 sq. rd. remaining. We write 25 sq. rd. in the quotient and reduce 50 sq. rd. to sq. yd. In 1 sq. rd. there are $30\frac{1}{4}$ sq. yd., and in 50 sq. rd. there are 50 times $30\frac{1}{4}$ sq. yd. $= 1,512\frac{1}{2}$ sq. yd. 62 is contained in $1,512\frac{1}{2}$ sq. yd. 24 times and $24\frac{1}{2}$ sq. yd. remaining. In 1 sq. yd. there are 9 sq. ft., and in $24\frac{1}{2}$ sq. yd. there are $24\frac{1}{2} \times 9 = 220\frac{1}{2}$ sq. ft. 62 is contained in $220\frac{1}{2}$ sq. ft. 3 times and $34\frac{1}{2}$ sq. ft. remaining. We write 3 sq. ft. in the quotient and reduce $34\frac{1}{2}$ sq. ft. to sq. in. In 1 sq. ft. there are 144 sq. in., and in $34\frac{1}{2}$ sq. ft. there are $34\frac{1}{2} \times 144 = 4,968$ sq. in. 62 is contained in 4,968 sq. in. 80 times and 8 sq. in. remaining.

We write 80 sq. in. in the quotient.

It should be borne in mind that it is only for the purpose of illustrating the method that this problem is carried out to square inches. It is not customary to reduce any lower than square rods in calculating the area of a farm.

ARITHMETIC.

(SECTION 5.)

(QUESTIONS 118–133.)

(118) To square a number, we must multiply the number by itself once, that is, use the number twice as a factor Thus, the second power of 108 is $108 \times 108 = 11{,}664$. Ans

$$\begin{array}{r} 108 \\ 108 \\ \hline 864 \\ 1080 \\ \hline 11664 \end{array}$$

(119) $9^5 = 9 \times 9 \times 9 \times 9 \times 9 = 59{,}049$. Ans.

$$\begin{array}{r} 9 \\ 9 \\ \hline 81 \\ 9 \\ \hline 729 \\ 9 \\ \hline 6561 \\ 9 \\ \hline 59049 \end{array}$$

(120) (*a*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

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Since there are four decimal places in the multiplicand and 4 in the multiplier, we must point off $4 + 4 = 8$ decimal places in the product; but as there are only 5 figures in the product, we prefix three ciphers to form the eight necessary decimal places in the first product.

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we must point off $8 + 4 = 12$ decimal places in the product; but as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

$$\begin{array}{r} .0133 \\ .0133 \\ \hline 399 \\ 399 \\ 133 \\ \hline .00017689 \\ .0133 \\ \hline 53067 \\ 53067 \\ 17689 \\ \hline .000002352637 \end{array}$$

(121) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(122) $\sqrt{90} = ?$ The root is evidently 9 plus an interminable decimal. Trying 9 for one factor, the other is $90 \div 9 = 10$, and the first approximation is $\frac{9 + 10}{2} = 9.5$. $90 \div 9.5 = 9.473+$, and the second approximation is $\frac{9.5 + 9.473}{2} = 9.486+$, or 9.49 to three figures. Using 9.49 for one factor, the other is $90 \div 9.49 = 9.48366+$, and the third approximation is $\frac{9.49 + 9.48366}{2} = 9.48683$ or $9.4868+$ to five figures. Ans.

This solution may be shortened by using the table and applying the method described in Art. **281** to find the first three significant figures of the root. Referring to the table, the first two significant figures of the root are 9.4; the first difference is $90.25 - 88.36 = 1.89$; the second difference is

$90 - 88.36 = 1.64$; $1.64 \div 1.89 = .86+$. Hence, the first three figures are 9.48.

(123) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer, and multiply the denominators together for the denominator of the answer.

$$\left(3\frac{3}{4}\right)^3 = \frac{15}{4} \times \frac{15}{4} \times \frac{15}{4} = \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3.375}{64} = 52\frac{47}{64}$$

$= 52.734375$. Ans.

$$3\frac{3}{4} = \frac{3 \times 4 + 3}{4} = \frac{12 + 3}{4} = \frac{15}{4}.$$

```
  15          64)3375(52 47/64
  15             320
  --             ---
  75              175
 15               128
 ---              ---
 225               47
  15
----
1125
225
----
3375
```

```
64)47.000000(.734375
   448
   ---
    220
    192
    ---
     280
     256
     ---
      240
      192
      ---
       480
       448
       ---
        320
        320
        ---
```

Since six ciphers were annexed to the dividend, six decimal places must be pointed off in the quotient.

(124) $\sqrt[3]{92,416} = ?$ Pointing off into periods, the result is 92′416. Since 92 lies between $4^3 = 64$ and $5^3 = 125$, the root is 4+. Trying 4 for one of the two equal factors, the third factor is $92 \div 4^2 = 92 \div 16 = 5.75$. Trying 5 for one of the two equal factors, the third factor is $92 \div 5^2 = 92 \div 25 = 3.68$. Difference between 4 and 5.75 is 1.75, and between 5 and 3.68 is 1.32; hence, use 5, the first approximation being $\frac{2 \times 5 + 3.68}{3} = 4.56$, or 4.6 to two figures.

Using 46 for one of the two equal factors, the third factor is $92416 \div 46^2 = 92416 \div 2116 = 43.67+$, and the second approximation is $\frac{2 \times 46 + 43.67}{3} = 45.22+$, or 45.2 to three figures.

Using 45.2 for one of the two equal factors, the third factor is $92416 \div 45.2^2 = 92416 \div 2043.04 = 45.2345+$, and the third approximation is $\frac{2 \times 45.2 + 45.2345+}{3} = 45.2115+$, or 45.212− to five figures. Ans.

The first three significant figures may also be found by the aid of the table and the method described in Art. **282.**

In order to obtain two figures of the root from the table, we place a decimal point between the first and second significant periods; the result is 92.416. Referring to the table, the first two figures of the root are 4.5; the first difference is $97.336 - 91.125 = 6.211$; the second difference is $92.416 - 91.125 = 1.291$; $1.291 \div 6.211 = .20+$. Therefore, $\sqrt[3]{92.416} = 4.52$ and $\sqrt[3]{92,416} = 45.2$ to three significant figures.

(125) $\sqrt{502,681} = ?$ Pointing off into periods, we have 50′26′81. The first figure of the root is evidently 7, since $7^2 = 49$ and $8^2 = 64$. The two factors then are 7, and $50 \div 7 = 7.14+$. The first approximation is $\frac{7 + 7.14+}{2} = 7.07+$, or 7.1 to two figures. To find the second approximation, we use the first two periods and drop the decimal point in the first approximation. One factor is then 71 and the other $5026 \div 71 = 70.78+$. The second approximation is therefore $\frac{71 + 70.78}{2} = 70.89$, or 70.9 to three figures. Using 709 for one factor, the other is $502681 \div 709 = 709$. Hence, the number is a perfect power and the root is 709. Ans.

Using the table and placing a decimal point between the first and second periods so that we may obtain two figures of the root from the table, the number becomes $\sqrt{50.2681}$ or to four figures, $\sqrt{50.27}$. Referring to the table, the first two figures of the root are 7.0; the first difference is $50.41 - 49.00 = 1.41$; the second difference is $50.27 - 49.00 = 1.27$; $1.27 \div 1.41 = .9+$. Therefore, $\sqrt{502{,}681} = 709$ to three figures

(126) $\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$. Ans.

(127) $4^3 = 4 \times 4 \times 4 = 64$.

$$\sqrt[3]{8} = 2.$$

$$4^3 - \sqrt[3]{8} = 64 - 2 = 62. \quad \text{Ans.}$$

(128) Since $\frac{3}{8} = .375$, $\sqrt[3]{\frac{3}{8}} = \sqrt[3]{.375}$. Moving the decimal point three places to the right, the number becomes 375. Since 375 lies between $7^3 = 343$ and $8^3 = 512$, the root is $7+$. Trying 7 for one of the two equal factors, the third factor is $375 \div 7^2 = 375 \div 49 = 7.65+$, and the first approximation is $\frac{2 \times 7 + 7.65}{3} = 7.21+$, or 7.2 to two figures. As the difference between the equal and unequal factors is very slight, it is not necessary to try 8.

Using 7.2 for one of the two equal factors, the third factor is $375 \div 7.2^2 = 375 \div 51.84 = 7.233+$, and the second approximation is $\frac{2 \times 7.2 + 7.233}{3} = 7.211+$, or 7.21 to three figures.

Using 7.21 for one of the two equal factors, the third factor is $375 \div 7.21^2 = 375 \div 51.9841 = 7.21374+$, and the third approximation is $\frac{2 \times 7.21 + 7.21374}{3} = 7.21124+$, or

7.2112+ to five figures. Since the number is entirely decimal, the root is wholly decimal; hence, locating the decimal point, the $\sqrt[3]{.375} = .72112+$. Ans.

By aid of the table, the first three figures are determined as follows: Move the decimal point three places to the right so it will fall between the first period 375 and the cipher period that follows.

Referring to the table, the first two figures of the root are 7.2; the first difference is 389. $17 - 373.248 = 15.769$; the second difference is $375.000 - 373.248 = 1.752$; $1.752 \div 15.769 = .11+$, or .1 to one figure. Hence, the first three figures are 7.21, or the $\sqrt[3]{.375} = .721$. The fourth and fifth figures are then determined as previously indicated.

(129) $\sqrt{.3364} = ?$

Moving the decimal point two places to the right, so that the first period may be integral, the result is 33.64. The first two factors are evidently 5 and $33 \div 5 = 6.6$, and the first approximation is $\frac{5 + 6.6}{2} = 5.8$. $33.64 \div 5.8 = 5.8$. Hence, the given number is a perfect power, and as it is wholly decimal, $\sqrt{.3364} = .58$. Ans.

(130) $\sqrt{3.1416} = ?$

Pointing off, we obtain 3.14′16. The first two significant figures are 3.1. It is evident that the first figure of the root is 1, since $1^2 = 1$ and $2^2 = 4$. Using 1 as one factor, the other is $3.1 \div 1 = 3.1$, and the first approximation is $\frac{1 + 3.1}{2} = 2.05$. Had 2 been used as one factor, the other would have been $3.1 \div 2 = 1.55$, and the first approximation would have been $\frac{2 + 1.55}{2} = 1.77+$, or 1.8 to two figures. In the first case, the difference between the two factors is $3.1 - 1 = 2.1$; in the second case, the difference is $2 - 1.55 = .45$. As the factors are more nearly equal in the second case than in the first, it is evident that 1.8 is more nearly equal to the correct value of the root than 2.05 is; hence, 1.8 will be used for the first approximation.

For the second approximation, use the first two periods and 1.8 for one factor, the other factor is $3.14 \div 1.8 = 1.74+$; hence, the second approximation $= \frac{1.8 + 1.74}{2} = 1.77$. Using 1.77 for one factor, the other is $3.1416 \div 1.77 = 1.77491+$. The third approximation is $\frac{1.77 + 1.77491}{2} = 1.772455$, or 1.7725– to five figures. Ans.

Using the table to find the first three figures, the first two figures of the root are 1.7; the first difference is $3.24 - 2.89 = .35$; the second difference is $3.14 - 2.89 = .25$; $.25 \div .35 = .71+$. Therefore, $\sqrt{3.1416} = 1.77$ to three significant figures.

(131) Since some number multiplied by itself equals 114.9184, then the number is $\sqrt{114.9184}$. Pointing off into periods and placing the decimal point between the first and second periods, we have 1.14'91'84. Considering the first two figures, it is evident that the first figure of the root is 1. Using 1 as one factor, the other is $1.1 \div 1 = 1.1$, and the first approximation is $\frac{1 + 1.1}{2} = 1.05$, or 1.1 to two figures. Using the first three figures and 1.1 for one factor, the other factor is $1.15 \div 1.1 = 1.045+$, and the second approximation is $\frac{1.1 + 1.045}{2} = 1.072+$, or 1.07 to three figures. Using 1.07 for one factor, the other is $1.149184 \div 1.07 = 1.074003+$, and the third approximation is $\frac{1.07 + 1.074003}{2} = 1.072001+$, or 1.0720 to five figures. Noticing that the square of the last significant figure is $2^2 = 4$, which corresponds to the last figure of the given number, and that the fifth and sixth figures of the third approximation are ciphers, we suspect that the given number is a perfect power. We find such to be the case on squaring 1.072. Since there are two periods in the integral part of the number, there are two figures in the integral part of the root, and $\sqrt{114.9184} = 10.72$. Ans.

Using the table to find the first three figures of the root, the first two figures of the root are 1.0; the first difference is $1.21 - 1.00 = .21$; the second difference is $1.15 - 1.00 = .15$; $.15 \div .21 = .7$. Therefore, the first three figures are 10.7.

(132) $\sqrt{3,486,784} = ?$

Pointing off into periods, we have 3′48′67′84. Placing a decimal point between the first and second periods and using the first two figures, we obtain 3.5. Considering 2 as one factor, the other is $3.5 \div 2 = 1.75$, and the first approximation is $\frac{2+1.75}{2} = 1.87+$, or 1.9 to two figures. For the second approximation, we use the first two periods and 19 for one factor, the other factor being $349 \div 19 = 18.37+$. We used 349 instead of 348 because the fourth figure was 6, and the number correct to three figures is 349. The second approximation is $\frac{19+18.37}{2} = 18.68+$, or 18.7 to three figures. $34868 \div 187 = 186.459+$; $\frac{187 + 186.459}{2} = 186.729+$, or 1867.3− to five figures. Ans.

In order to obtain three figures from the table, we place the decimal point between the first and second periods, and use the first two periods only; that is, we find the value of $\sqrt{3.49}$. Referring to the table, the first two figures of the root are 1.8; the first difference is $3.61 - 3.24 = .37$; the second difference is $3.49 - 3.24 = .25$; $.25 \div .37 = .67+$, or .7 to one figure. Therefore, $\sqrt{3,486,784} = 187$ to three significant figures.

The fourth and fifth figures may be found as previously indicated.

(133) $\sqrt{.00041209} = ?$

Pointing off into periods, the result is .00′04′12′09. Placing the decimal point between the first and second periods of the significant part of the number, we obtain 4.1 for the first two figures. The first factor is evidently 2 and the second factor $4.1 \div 2 = 2.05$. The first approximation is $\frac{2+2.05}{2} = 2.02+$, or 2.0 to two figures. Using 20 for one factor, the other is

$412 \div 20 = 20.6$, and the second approximation is $\frac{20 + 20.6}{2} = 20.3$.

Using 203 for one factor, the other is $41209 \div 203 = 203$. Hence the number is a perfect power and the significant figures of the root are 203. There being one full cipher period following the decimal point, the root is .0203. Ans.

Using the table, we find that the first two figures of the root are 2.0; the first difference is $4.41 - 4.00 = .41$; the second difference is $4.12 - 4.00 = .12$; $.12 \div .41 = .3$, nearly Therefore, $\sqrt{.00041209} = .0203$ to three significant figures.

ARITHMETIC.

(SECTION 6.)

(QUESTIONS 143–167.)

(143) $11.7 : 13 :: 20 : x.$

$11.7\,x = 13 \times 20$

$11.7\,x = 260$

The product of the means equals the product of the extremes.

$$x = \frac{260}{11.7}\;\begin{array}{l})\,260.000\,(\,22.22+ \quad \text{Ans.} \\ \;\;234 \\ \hline \;\;\;\;260 \\ \;\;\;\;234 \\ \hline \;\;\;\;\;\;260 \\ \;\;\;\;\;\;234 \\ \hline \;\;\;\;\;\;\;\;260 \\ \;\;\;\;\;\;\;\;234 \\ \hline \;\;\;\;\;\;\;\;\;\;26 \end{array}$$

(144) (*a*) $20 + 7 : 10 + 8 :: 3 : x.$

$27 : 18 :: 3 : x$

$27\,x = 18 \times 3$

$27\,x = 54$

$x = \frac{54}{27} = 2.$ Ans.

(*b*) $12^2 : 100^2 :: 4 : x.$

$144 : 10{,}000 :: 4 : x$

$144\,x = 10{,}000 \times 4$

$144\,x = 40{,}000$

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$x = \frac{40,000}{144}$

```
144 ) 40000.0 ( 277.7+   Ans
      288
      1120
      1008
       1120
       1008
        1120
        1008
         112
```

(145) (*a*) $\frac{4}{x} = \frac{7}{21}$, is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \frac{84}{7} \text{ or } 12. \quad \textbf{Ans.}$$

(*b*) In like manner,

$\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16$

$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \frac{192}{16} = 12. \quad \textbf{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x : 100$.

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to

$15 : 45 :: 60 : x$.

$15x = 45 \times 60$

$15x = 2,700$

$x = \frac{2,700}{15} = 180.$ **Ans.**

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to

$10 : 150 :: x : 600$.

$150x = 10 \times 600$

$150x = 6,000$

$x = \frac{6,000}{150} = 40.$ **Ans.**

(146) $x : 5 :: 27 : 12.5.$

$$\begin{array}{r} 5 \\ \hline 12.5\,)\,1\,3\,5.0\,(\,10\frac{4}{5} \text{ Ans.} \\ 1\,2\,5 \\ \hline 1\,0\,0 \end{array} \qquad \frac{100}{125} = \frac{4}{5}$$

(147) $45 : 60 :: x : 24$

$$60\,x = 45 \times 24$$
$$60\,x = 1{,}080$$
$$x = \frac{1{,}080}{60} = 18. \text{ Ans.}$$

(148) $x : 35 :: 4 : 7.$

$$7\,x = 35 \times 4$$
$$7\,x = 140$$
$$x = \frac{140}{7} = 20. \quad \text{Ans.}$$

(149) $9 : x :: 6 : 24.$

$$6\,x = 9 \times 24$$
$$6\,x = 216$$
$$x = \frac{216}{6} = 36. \quad \text{Ans.}$$

(150)

$\sqrt[3]{1{,}000} : \sqrt[3]{1{,}331} :: 27 : x.$

$10 : 11 :: 27 : x.$

$$10\,x = 297.$$
$$x = \frac{297}{10} = 29.7. \quad \text{Ans.}$$

$\sqrt[3]{1{,}000} = 10.$

$\sqrt[3]{1{,}331} = 11.$

$$\begin{array}{rrr} 1 & 1 & 1'3\,3\,1\,(\,11 \\ 1 & 2 & 1 \\ \hline 2 & 3\,0\,0 & 3\,3\,1 \\ 1 & 3\,1 & 3\,3\,1 \\ \hline 3\,0 & 3\,3\,1 & \\ 1 & & \\ \hline 3\,1 & & \end{array}$$

(151) $64 : 81 = 21^2 : x^2.$

Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} = \sqrt{21^2} : \sqrt{x^2}$ is the same as

$$8 : 9 = 21 : x$$
$$8\,x = 189$$
$$x = 23.625. \quad \text{Ans.}$$

(152) $7 + 8 : 7 = 30 : x$ is equivalent to

$$15 : 7 = 30 : x.$$
$$15\,x = 7 \times 30$$
$$15\,x = 210$$
$$x = \frac{210}{15} = 14. \quad \text{Ans.}$$

(153) 2 ft. 5 in. = 29 in.; 2 ft. 7 in. = 31 in. Stating as a direct proportion, 29 : 31 = 2,480 : x. Now, it is easy to see that x will be greater than 2,480. But x should be less than 2,480, since, when a man lengthens his steps, the number of steps required for the same distance is less; hence, the proportion is an inverse one, and

$$29 : 31 = x : 2{,}480,$$
or, $31x = 71{,}920;$
whence, $x = 71{,}920 \div 31 = 2{,}320$ steps. Ans.

(154) This is evidently a direct proportion. 1 hr. 36 min. = 96 min.; 15 hr. = 900 min. Hence,

$$96 : 900 = 12 : x,$$
or, $96x = 10{,}800;$
whence, $x = 10{,}800 \div 96 = 112.5$ mi. Ans.

(155) This is also a direct proportion; hence,

$$27.63 : 29.4 = .76 : x,$$
or, $27.63x = 29.4 \times .76 = 22.344;$
whence, $x = 22.344 \div 27.63 = .808 +$ lb. Ans.

(156) 2 gal. 3 qt. 1 pt. = 23 pt.; 5 gal. 3 qt. = 46 pt. Hence,

$$23 : 46 = 5 : x,$$
or, $23x = 46 \times 5 = 230;$
whence, $x = 230 \div 23 = 10$ days. Ans.

(157) Stating as a direct proportion, and squaring the distances, as directed by the statement of the example, $6^2 : 12^2 = 24 : x$. Inverting the second couplet, since this is an inverse proportion,

$$6^2 : 12^2 = x : 24.$$

Dividing both terms of the first couplet (see Art. **310**) by 6

$1^2 : 2^2 = x : 24$; or $1 : 4 = x : 24$;
whence, $4x = 24$, or $x = 6$ degrees. Ans.

(158) Taking the dimensions as the causes,

$$\begin{array}{r|ll|l} \cancel{12} & 15 & & \\ \cancel{4} & 5 & & \\ 2 & \cancel{2} & = \cancel{12} & x, \text{ whence, } 2x = 75, \text{ or, } x = \$37.50. \\ \cancel{8} & \cancel{6} & & \qquad\qquad\qquad\qquad\qquad \text{Ans.} \end{array}$$

(159) 2 hr. = 120 min.; 14 hr. 28 min. = 868 min.

Hence, $120 : 868 = 100 : x$,

or, $120x = 86{,}800$;

whence, $x = 723\frac{1}{3}$ gal. Ans.

(160) Taking the dimensions as the causes,

$$\begin{array}{r|ll|l} \cancel{14} & \cancel{2} & & \\ \cancel{28} & \cancel{20} & & \\ 2 & & = \cancel{798} & x, \text{ whence, } 2x = 17 \times 57 = 969, \\ \cancel{12} & 17 & \quad 57 & \qquad\qquad \text{or, } \quad x = 484\tfrac{1}{2} \text{ bbl.} \quad \text{Ans} \\ \cancel{10} & \cancel{6} & & \end{array}$$

(161) 8 hr. 40 min. = 520 min. Hence,

$$444 : 1{,}060 = 520 : x,$$

$$\text{or, } x = \frac{1{,}060 \times \overset{130}{\cancel{520}}}{\underset{111}{\cancel{444}}} = \frac{\mathbf{137{,}800}}{\mathbf{111}} = 1{,}241.44 + \text{ min.} = 20 \text{ hr. } 41.44 + \text{ min.} \quad \text{Ans.}$$

(162) 1 min. = 60 sec. Hence,

$$5\tfrac{1}{2} : 60 = 6{,}160 : x,$$

$$\text{or, } x = \frac{60 \times 6{,}160}{5.5} = 67{,}200 \text{ ft.} \quad \text{Ans.}$$

(163) Writing the statement as a direct proportion, $8 : 10 = 5 : x$, it is easy to see that x will be greater than 5; but, it should be smaller, since by working longer hours, fewer men will be required to do the same work. Hence, the proportion is inverse. Inverting the second couplet,

$$8 : 10 = x : 5,$$

$$\text{or,} \quad x = \frac{\overset{4}{\cancel{8}} \times \cancel{5}}{\underset{2}{\cancel{10}}} = 4 \text{ men.} \quad \text{Ans.}$$

(164) Taking the times as the causes,

$$\begin{array}{r|rr|l} \cancel{20} & & \cancel{25} & 14 \\ & & \cancel{5} & \cancel{70} \\ & & =\cancel{540} & \cancel{630}; \text{ whence, } 3x = 2 \times 14 = 28, \text{ or } x = 9\tfrac{1}{3} \text{ hr.} \\ \cancel{10} & x & \cancel{27} & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{Ans.} \\ 2 & & 3 & \end{array}$$

(165) Taking the horsepowers as the effects, we have for the known causes in example 4, Art. **349**, 14^2, 500, and 48, and for the known effect 112 horsepower. Hence,

$$\begin{array}{r|l|l} 14^2 & 30^2 & \\ 500 & 660 = 112 & x, \text{ or} \\ 48 & 42 & \end{array} \quad \begin{array}{r|ll|l} & 9 & & \\ \cancel{14} & \cancel{900} & & \\ \cancel{196} & 22 & & \\ \cancel{5} & \cancel{110} & & \\ \cancel{500} & \cancel{660} = & \cancel{112} & x; \\ \cancel{6} & 3 & \cancel{8} & \\ \cancel{48} & \cancel{42} & & \end{array}$$

whence, $x = 9 \times 22 \times 3 = 594$ horsepower. Ans.

(166) First find the volume of the cylinder in cubic inches, as in the example, Art. **345.** The volume, multiplied by the weight of one cubic inch (.261 lb.), will evidently be the weight of the cylinder. Hence,

$$\begin{array}{r|l|l} 10^2 & 12^2 & \\ & = 1{,}570.8 & x, \text{ or} \\ 20 & 60 & \end{array} \quad \begin{array}{r|l|l} 100 & 144 & \\ & 3 \quad = 1.570.8 & x; \\ \cancel{20} & \cancel{60} & \end{array}$$

whence, $x = \dfrac{144 \times 3 \times 1{,}570.8}{100} = 6{,}785.856$ cu. in. Therefore weight of cylinder $= 6{,}785.856 \times .261 = 1{,}771.11$ lb. Ans

(167) Referring to the example in Art. **348,**

$$\begin{array}{r|l|l} 15 & 40 & \\ & & \\ 20^2 & 18^2 = 187 & x, \text{ or} \\ 10 & 12 & \end{array} \quad \begin{array}{r|l|l} & \cancel{4} & \\ 5 & \cancel{40} & \\ \cancel{15} & & \\ 100 & & \\ \cancel{400} & 324 = 187 & x; \\ \cancel{10} & 4 & \\ & \cancel{12} & \end{array}$$

whence, $x = \dfrac{324 \times 4 \times 187}{500} = 484.7$ lb. Ans.

FORMULAS.

(QUESTIONS 168–176.)

(168) Substituting for D, x, B, and i their values,

$$C = \frac{D - x}{B + i} = \frac{120 - 12}{10 + 3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line, or numerator, is to be divided by the one below the line, or denominator.

(169) Substituting for A, h, D, and x their values,

$$\frac{A\,h + D}{2\,x + 6} = \frac{(5 \times 200) + 120}{(2 \times 12) + 6} = \frac{1{,}000 + 120}{24 + 6} = \frac{1{,}120}{30} = 37\tfrac{1}{3}.$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(170) Substituting for B, h, A, x, and i their values,

$$r = \frac{3.246 \times B \times h}{\frac{A\,x + h}{A\,i - B}} = \frac{3.246 \times 10 \times 200}{\frac{(5 \times 12) + 200}{(5 \times 3.5) - 10}} = \frac{6{,}492}{\frac{260}{7.5}} =$$

$$6{,}492 \div \frac{260}{7.5} = 6{,}492 \times \frac{7.5}{260} = 187.269+. \quad \text{Ans.}$$

(171) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B + 1.5}} = \sqrt{\frac{5 \times 120}{(3.5 \times 10) + 1.5}} = \sqrt{\frac{600}{36.5}} =$$

$$\sqrt{16.4383} = 4.05+. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(172) Substituting for B, x, h, and A their values,

$$u = \sqrt[3]{\frac{B\,x}{.00018\,h\,(A^2-x)}} = \sqrt[3]{\frac{10 \times 12}{.00018 \times 200 \times (5^2 - 12)}} =$$

$$\sqrt[3]{\frac{120}{.036 \times (25 - 12)}} = \sqrt[3]{\frac{120}{.036 \times 13}} = \sqrt[3]{\frac{120}{.468}} =$$

$$\sqrt[3]{256.41+} = 6.35+. \quad \text{Ans.}$$

(173) Substituting for h, D, and A their values,

$$f = \frac{10\,(h-D)^2}{\sqrt[3]{D+A}} = \frac{10\,(200-120)^2}{\sqrt[3]{120+5}} = \frac{10 \times 80^2}{\sqrt[3]{125}} = \frac{64{,}000}{5} = 12{,}800. \quad \text{Ans.}$$

(174) Substituting for B, A, and D their values,

$$g = \frac{(B-A)^2 - \sqrt[3]{D+A}}{A^3 - (1+D)} = \frac{(10-5)^2 - \sqrt[3]{120+5}}{5^3 - (1+120)} =$$

$$\frac{5^2 - \sqrt[3]{125}}{125 - 121} = \frac{25-5}{4} = \frac{20}{4} = 5. \quad \text{Ans.}$$

(175) Substituting for A, B, and h their values,

$$k = \sqrt{\frac{A\,B^2}{\sqrt[3]{A\,h}}} = \sqrt{\frac{5 \times 10^2}{\sqrt[3]{5 \times 200}}} = \sqrt{\frac{5 \times 100}{\sqrt[3]{1{,}000}}} = \sqrt{\frac{500}{10}} =$$

$$\sqrt{50} = 7.071+. \quad \text{Ans.}$$

(176) Substituting for A, h, D, x, and B their values,

$$T = \sqrt{\frac{A^2\left[490 + \frac{(h\,x)^2}{D^2}\right]}{h + \frac{x}{D}(A^2 - B)^2}} = \sqrt{\frac{5^2\left[490 + \frac{(200 \times 12)^2}{120^2}\right]}{200 + \frac{12}{120}(5^2 - 10)^2}} =$$

$$\sqrt{\frac{25\,(490 + 400)}{200 + (\frac{1}{10} \times 225)}} = \sqrt{\frac{25 \times 890}{200 + 22.5}} = \sqrt{\frac{22{,}250}{222.5}} =$$

$$\sqrt{100} = 10. \quad \text{Ans.}$$

GEOMETRY AND TRIGONOMETRY.

(1) When one straight line meets another straight line at a point between the ends, the sum of the two adjacent angles equals two right angles. Therefore, since one of the angles equals $\frac{4}{5}$ of a right angle, the other angle equals two right angles (or $\frac{10}{5}$) minus $\frac{4}{5}$. We have, then, $\frac{10}{5} - \frac{4}{5} = \frac{6}{5}$, or $1\frac{1}{5}$ right angles. Ans.

(2) It is an isosceles triangle, since the sides opposite the equal angles are equal.

(3) A regular decagon has 10 equal sides; therefore, the length of one side is $40 \div 10 = 4$ in. Ans.

(4) The sum of all the interior angles of any polygon equals two right angles, multiplied by the number of sides in the polygon less two. As a regular dodecagon has 12 equal sides, the sum of the interior angles equals two right angles × 10 $(= 12 - 2)$, or 20 right angles. Since there are 12 equal angles, the size of any one of them equals $20 \div 12$, or $1\frac{2}{3}$ right angles. Ans.

(5) Equilateral triangle.

(6) Since the two angles *A* and *C*, Fig. I, are equal, the triangle is isosceles, and a line drawn from the vertex *B* will bisect the line *A C*, the length of which is 7 inches; therefore,

$$AD = DC = 7 \div 2 = 3\tfrac{1}{2} \text{ in. Ans.}$$

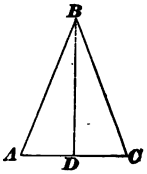

FIG. I.

(7) The length of the line $= \sqrt{12^2 - 9^2} + \sqrt{15^2 - 9^2}$, or 19.94 in. Ans.

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(8) One of the angles of an equiangular octagon is equal to $\frac{1}{8}$ of 12 right angles, or $1\frac{1}{2}$ right angles, since the sum of the interior angles of the equiangular octagon equals 12 right angles.

(9) See Art. **71.**

(10) In Fig. II, $AB = 4$ inches, and $OA = 6$ inches. We first find the length of OD. $OD = \sqrt{\overline{OA}^2 - \overline{DA}^2}$; but $\overline{OA}^2 = 6^2$, or 36, and $\overline{DA}^2 = \left(\frac{4}{2}\right)^2$, or 4; therefore, $OD = \sqrt{36 - 4}$, or 5.657.

$DC = OC - OD$, or $DC = 6 - 5.657$, or .343 inch. In the right triangle ADC, we have AC, which is the chord of half the arc ACB, equals $\sqrt{2^2 + .343^2}$, or 2.03 in. Ans.

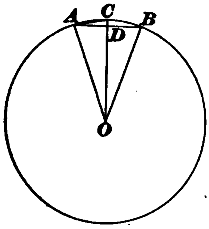

FIG. II.

(11) Given, $OC = 5\frac{3}{4}$ inches and $OA = 17 \div 2 = 8\frac{1}{2}$ inches, to find AB (see Fig. III). AC, which is one-half the chord AB, equals $\sqrt{\overline{OA}^2 - \overline{OC}^2}$; therefore,

$AC = \sqrt{(8\frac{1}{2})^2 - (5\frac{3}{4})^2}$, or 6.26 inches.

Now, $AB = 2 \times AC$; therefore, $AB = 2 \times 6.26$, or 12.52 in. Ans.

(12) The arc intercepted equals $\frac{3}{4}$ of 4, or 3 quadrants. As the inscribed angle is measured by one-half the intercepted arc, we have $3 \div 2 = 1\frac{1}{2}$ quadrants as the size of the angle.

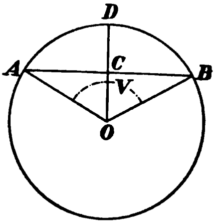

FIG. III.

(13) Four right angles $\div \frac{2}{7} = 4 \times \frac{7}{2}$, or 14 equal sectors.

(14) Since 24 inches equals the perimeter, we have $24 \div 8 = 3$ inches, as the length of each side or chord.

Then, $\sqrt{\left(\frac{3}{2}\right)^2 + 3.62^2} = 3.92$ in., radius.

$3.92 \times 2 = 7.84$ in., diameter. Ans.

(15) In 19° 19′ 19″ there are 69,559 seconds, and in 360°, or a circle, there are 1,296,000 seconds. Therefore, 69,559 seconds equal $\frac{69,559}{1,296,000}$, or .053672 part of a circle. Ans.

(16) Referring to Fig. 75 of the text and using the values given in the example, we have $AB = 26$ feet 7 inches, or 26.583 feet; $AC = 40$ feet; and the included angle $A = 36° \; 20' \; 43''$. Then in the right triangle ADB, AB is known, and also the angle A.

Hence, by rule **1**, Art. **98**, $BD = 26.583 \times \sin 36° \; 20' 43'' = 26.583 \times .59265 = 15.754$ feet. By rule **3**, Art. **98**, $AD = 26.583 \times \cos 36° \; 20' \; 43'' = 26.583 \times .80546 = 21.411$. $AC - AD = 40 - 21.411 = 18.589$ feet $= DC$. In the right triangle CDB, the two sides BD and DC are known; hence, $\tan C = \frac{BD}{DC} = \frac{15.754}{18.589} = .84749$, and angle $C = 40° \; 16' \; 52''$. Ans.

Applying rule **2**, Art. **98**, $BC = \frac{BD}{\sin C} = \frac{15.754}{\sin 40° \; 16' \; 52''} = \frac{15.754}{.64654} = 24.37$, or 24 ft. 4.4 in. Ans.

Angle $B = 180° - (36° \; 20' \; 43'' + 40° \; 16' \; 52'') = 180° - 76° \; 37' \; 36'' = 103° \; 22' 25''$. Ans.

(17) See Fig. 76 of the text. Solving the triangle ABC, we first find BD. By rule **1**, Art. **98**, $BD = AB \times \sin A = 16\tfrac{5}{12} \times \sin \; 54° \; 54' \; 54'' = 16\tfrac{5}{12} \times .81830 = 13.434$ feet. Sin $BCD = \frac{BD}{BC} = \frac{13.434}{13.542} = .99202$; whence, angle $BCD = 82° \; 45' \; 30''$. Ans.

By rule **3**, $CD = BC \times \cos C = 13\tfrac{13}{24} \times \cos \; 82° \; 45' \; 30'' = 13\tfrac{13}{24} \times .12605 = 1.7069$ feet.

By rule **3**, $AD = AB \times \cos A = 16\tfrac{5}{12} \times \cos 54° \; 54' \; 54'' = 16\tfrac{5}{12} \times .57479 = 9.4361$ feet.

In the triangle ABC, the angle ACB is the supplement (see Art. **27**) of the angle BCD and equals 180° − angle BCD, or $ACB = 180° - 82° \; 45' \; 30'' = 97° \; 14' \; 30''$. Ans.

Angle $ABC = 180° - (\text{angle } BAC + \text{angle } ACB) = 180° - (54° \; 54' \; 54'' + 97° \; 14' \; 30'') = 180° - 152° \; 9' \; 24'' = 27° \; 50' \; 36''$. Ans.

Side $AC = AD - CD$, or $AC = 9.4361 - 1.7069 = 7.7292$ $= 7$ ft. $8\frac{3}{4}$ in. Ans.

For the triangle ABC', angle $C' = BCD$ (isosceles triangle), or $C' = 82° 45' 30''$.

Angle $ABC' = 180° -$ (angle A + angle C') $= 180° - (54° 54' 54'' + 82° 45' 30'') = 180° - 137° 40' 24'' = 42° 19' 36''$. Ans.

$AC' = AD + C'D = 9.4361 + 1.7069 = 11.143 = 11$ ft. $1\frac{3}{4}$ in. Ans.

(18) If one-third of a certain angle equals $14° 47' 10''$, then the angle must be $3 \times 14° 47' 10''$, or $44° 21' 30''$. $2\frac{1}{2} \times 44° 21' 30''$, or $110° 53' 45''$, equals one of the other two angles. The third angle equals $180° - (110° 53' 45'' + 44° 21' 30'')$, or $24° 44' 45''$.

(19) Referring to Fig. 70 of the text, let $BC = 437$ feet and $AC = 792$ feet, to find the hypotenuse AB and the angles A and B.

$AB = \sqrt{\overline{AC}^2 + \overline{BC}^2} = \sqrt{\overline{792}^2 + \overline{437}^2} = \sqrt{818{,}233} = 904$ ft. $6\frac{3}{4}$ in. Ans.

By rule **4,** Art. **98,** $\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$, or $\tan A = \frac{437}{792} = .55177$; therefore, $A = 28° 53' 19''$. Ans.

Angle $B = 90° - 28° 53' 19''$, or $61° 6' 41''$. Ans.

(20) See Fig. IV. Angle $B = 180° - (29° 21' + 76° 44' 18'') = 180° - 106° 5' 18'' = 73° 54' 42''$.

From C, draw CD perpendicular to AB.

$AD = AC \cos A = 31.833 \times \cos 29° 21' = 31.833 \times .87164 = 27.747$ feet. $CD = AC \sin A = 31.833 \times \sin 29° 21' = 31.833 \times .49014 = 15.603$.

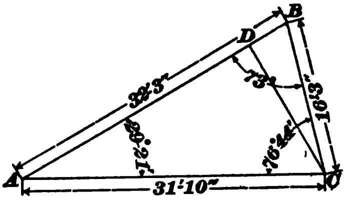

FIG. IV.

$$BC = \frac{CD}{\sin B} = \frac{15.603}{\sin 73° 54' 42''} = 16.24 \text{ feet} = 16 \text{ ft. } 3 \text{ in.}$$

$$DB = \frac{CD}{\tan B} = \frac{15.603}{\tan 73° 54' 42''} = 4.5 \text{ feet,}$$

$A\,B = A\,D + D\,B = 27.747 + 4.5 = 32.247 = 32$ feet, 3 inches.

$$\text{Ans.}\begin{cases} B\,C = 16 \text{ ft. } 3 \text{ in.} \\ A\,B = 32 \text{ ft. } 3 \text{ in.} \\ B = 73^\circ\ 54'\ 42''. \end{cases}$$

(21) By rule, Art. **135,**

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{89.42}{.7854}} = \sqrt{113.8528}, \text{ or } 10.67 \text{ in. Ans.}$$

Circumference equals 10.67×3.1416, or 33.52 in. Ans.

In a regular hexagon inscribed in a circle, each side is equal to the radius of the circle; therefore, $\frac{10.67}{2} = 5.335$ in. is the length of a side. Ans.

(22) Angle $m\,O\,B = \frac{1}{16}$ of 360°, or $22\frac{1}{2}^\circ$. $m\,O = \frac{1}{2}$ of $m\,n = \frac{1}{2}$ of 2, or 1 inch. See Fig. V.

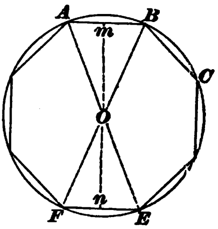

FIG. V.

Side $m\,B = O\,m \times \tan 22\frac{1}{2}^\circ$, or $m\,B = 1 \times .41421 = .41421$.

$A\,B = 2\,m\,B$; therefore, $A\,B = .82842$ inch.

Area of $A\ O\ B = \frac{1}{2} \times .82842 \times 1 = .41421$ square inch, which, multiplied by 8, the number of equal triangles, equals 3.31368 square inches.

Volume of bar $= 3.31368 \times 10 \times 12 = 397.6416$ cu. in.

Weight of bar equals $397.6416 \times .282 = 112.1349$, or 112 lb. 2 oz. Ans.

(23) $.5236 \times 16^3 = 2{,}144.66$ cubic inches equals the volume of a sphere 16 inches in diameter.

$.5236 \times 12^3 = 904.78$ cubic inches equals the volume of a sphere 12 inches in diameter.

The difference of the two volumes equals the volume of the spherical shell, and this multiplied by the weight per cubic inch equals the weight of the shell. Hence, we have $(2{,}144.66 - 904.78) \times .261 = 323.61$ lb. Ans.

(24) The circumference of the circle equals $\frac{5\frac{13}{32} \times 360}{27}$, or 72.0833 inches. The diameter, therefore, is $\frac{72.0833}{3.1416}$, or 22.95 in., nearly. Ans.

(25) (*a*) $17\frac{1}{64}$ inches = 17.016 inches.

Area of circle $= .7854d^2 = .7854 \times 17.016^2 = 227.41$ sq. in. Ans.

(*b*) $16° 7' 21'' = 16.1225°$. By rule, Art. **132,**

$$l = \frac{\pi d n}{360} = \frac{3.1416 \times 17.016 \times 16.1225}{360} = 2.394 \text{ in.} \quad \text{Ans.}$$

(26) (*a*) By rule, Art. **144,** area $= 12 \times 8 \times .7854 = 75.4$ sq. in. Ans.

(*b*) Applying the formula, Art. **143,** $a = 6$, $b = 4$, $D = \frac{6-4}{6+4} = \frac{2}{10} = \frac{1}{5}$.

$$\text{Perimeter} = 3.1416\,(6 + 4)\,\frac{64 - 3(\frac{1}{5})^4}{64 - 16(\frac{1}{5})^2} = 31.731 \text{ in.} \quad \text{Ans.}$$

(27) Area of base $= .7854 \times 7^2 = 38.484$ square inches.

Slant height of cone $= \sqrt{11^2 + 3\frac{1}{2}^2}$, or 11.5434 inches.

Circumference of base $= 7 \times 3.1416 = 21.9912$ inches.

Convex area of cone $= 21.9912 \times \frac{11.5434}{2} = 126.927$ square inches.

Total area $= 126.927 + 38.484 = 165.41$ sq. in. Ans.

(28) Volume of sphere equals $.5236 \times 10^3 = 523.6$ cubic inches.

Area of base of cone $= .7854 \times 10^2 = 78.54$ square inches.

$\frac{3 \times 523.6}{78.54} = 20$ in., altitude of cone. Ans.

(29) Volume of sphere $= .5236 \times 12^3 = 904.7808$ cubic inches.

Area of base of cylinder $= .7854 \times 12^2 = 113.0976$ square inches.

Height of cylinder $= \frac{904.7808}{113.0976} = 8$ in. Ans.

(30) (*a*) Area of the triangle equals $\frac{AC \times BD}{2}$, or $\frac{9\frac{1}{2} \times 12}{2} = 57$ sq. in. Ans.

(*b*) See Fig. VI. Angle $BAD = 79° 22'$; angle $ABD = 90° - 79° 22' = 10° 38'$. Side $AB = BD \div \sin 79° 22' = 12 \div .98283 = 12.209$ inches.

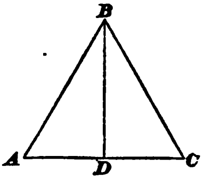

FIG. VI.

Side $AD = BD \times \tan 10° 38' = 12 \times .18775 = 2.253$ inches.

Side $DC = AC - AD = 9.5 - 2.253 = 7.247$ inches.

Side $BC = \sqrt{\overline{DB}^2 + \overline{DC}^2} = \sqrt{12^2 + 7.247^2} = \sqrt{196.519} = 14.018$ inches.

Perimeter of triangle equals $AB + BC + AC = 12.209 + 14.018 + 9.5 = 35.73$ in. Ans.

(31) The diagonal divides the trapezium into two triangles; the sum of the areas of these two triangles equals the area of the trapezium, which is, therefore,

$$\frac{11 \times 7}{2} + \frac{11 \times 4\frac{1}{4}}{2} = 61\tfrac{1}{8} \text{ sq. in. Ans.}$$

(32) Referring to Fig. III, example 11, we have OA or $OB = 10 \div 2 = 5$ inches, and $AB = 6\frac{3}{4}$ inches.

$\sin COB = \dfrac{CB}{OB} = \dfrac{6\frac{3}{4} \div 2}{5} = .67500$; therefore, angle $COB = 42° 27' 14.3''$.

Angle $AOB = (42° 27' 14.3'') \times 2 = 84° 54' 28.6''$. Ans.

$OC = OB \times \cos COB = 5 \times .73782 = 3.6891$.

Area of sector $= 10^2 \times .7854 \times \dfrac{84° 54' 28.6''}{360°} = 18.524$ square inches.

Area of triangle $= \dfrac{6.75 \times 3.6891}{2} = 12.450$ square inches.

$18.524 - 12.450 = 6.074$ sq. in., the area of the segment. Ans.

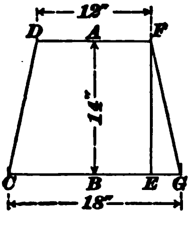

FIG. VII.

(33) See Fig. VII. Area of lower base $= 18^2 \times .7854 = 254.4696$ square inches.

Area of upper base $= 12^2 \times .7854 = 113.0976$ square inches.

$EG = BG - AF = 9 - 6$, or 3 inches.

Slant height $FG = \sqrt{\overline{EG}^2 + \overline{EF}^2} = \sqrt{3^2 + 14^2} = 14.32$ inches.

By formula, Art. **169**,

$$C = \frac{(p+p')s}{2} = \frac{37.6992 + 56.5488}{2} \times 14.32 = 674.8156 \text{ sq. in.}$$

Total area $= 674.8157 + 254.4696 + 113.0976 = 1{,}042.38$ sq. in. Ans.

By formula, Art. **170,** $V = (A + a + \sqrt{A \times a})\frac{h}{3} = (113.0976 + 254.4696 + \sqrt{113.0976 \times 254.4696})\ \frac{14}{3} = 2{,}506.997$ cu. in. Ans.

(34) Area of surface of sphere 27 inches in diameter $= \pi d^2 = 3.1416 \times 27^2 = 2{,}290.2$ sq. in. Ans.

(35) Area of end $= 19^2 \times .7854 = 283.5294$ square inches. Volume $= 283.5294 \times 24 = 6{,}804.7056$ cu. in. $= 3.938$ cu. ft. Ans.

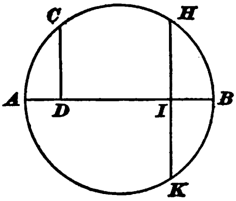

FIG. VIII.

(36) Given, $IB = 2$ inches and $HI = IK = \frac{14}{2} = 7$ inches to find the radius. See Fig. VIII.

$IB : HI = HI : AI$, or $2 : 7 = 7 : AI$.

Therefore, $AI = \frac{49}{2} = 24\frac{1}{2}$ inches.

$AB = AI + IB = 24\frac{1}{2} + 2 = 26\frac{1}{2}$ inches.

Radius $= \frac{AB}{2} = \frac{26\frac{1}{2}}{2} = 13\frac{1}{4}$ in. Ans.

(37) (*a*) In Fig. IX, given $OB = \frac{16}{2}$ or 8 inches, and $OA = 13 \div 2 = 6\frac{1}{2}$ inches, to find the volume, area, and weight.

Radius of center circle equals $\frac{8 + 6.5}{2}$, or $7\frac{1}{4}$ inches. Length of center line $= 3.1416 \times 2 \times 7\frac{1}{4} = 45.5532$ inches. The radius of the inner circle is $6\frac{1}{2}$ inches, and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, the area of the imaginary cross-section is $(1\frac{1}{2})^2 \times .7854 = 1.76715$ sq. in.

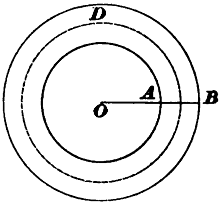

FIG. IX.

Volume of ring $= 1.76715 \times 45.5532 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.4993 \times .261 = 21$ lb. Ans.

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(38) The convex area $= 4 \times 5\frac{1}{4} \times 18 = 378$ sq. in. Ans.
Area of the bases $= 5\frac{1}{4} \times 5\frac{1}{4} \times 2 = 55.125$ square inches.
Total area $= 378 + 55.125 = 433.125$ sq. in. Ans.
Volume $= (5\frac{1}{4})^2 \times 18 = 496.125$ cu. in. Ans.

(39) In Fig. X, $OC = \frac{AC}{\tan 30°}$. ($\frac{1}{6}$ of $360° = 60°$, and since $AOC = \frac{1}{2}$ of AOB, $AOC = 30°$.)

$$OC = \frac{6}{.57735} = 10.392.$$

$$\text{Area of } AOB = \frac{12 \times 10.392}{2}$$

$$= 62.352 \text{ square feet.}$$

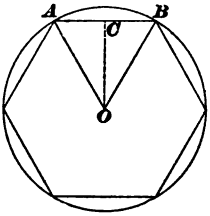

FIG. X.

Since there are 6 equal triangles in a hexagon, then the area of the base $= 6 \times 62.352$, or 374.112 square feet.

Perimeter $= 6 \times 12$, or 72 feet.

$$\text{Convex area} = \frac{72 \times 37}{2} = 1{,}332 \text{ sq. ft. Ans.}$$

Total area $= 1{,}332 + 374.112 = 1{,}706.112$ sq. ft. Ans.

(40) Area of the base $= 374.112$ square feet, and altitude $= 37$ feet. Since the volume equals the area of the base multiplied by $\frac{1}{3}$ of the altitude, we have

$$\text{volume} = 374.112 \times \frac{37}{3} = 4{,}614 \text{ cu. ft. Ans.}$$

(41) Given, $AB = 6\frac{7}{8}$ inches, and $OB = OA = 10 \div 2 = 5$ inches (see Fig. XI), to find the area of the sector.

Area of circle $= 10^2 \times .7854 = 78.54$ square inches.

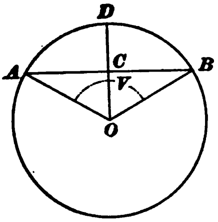

FIG. XI.

$$\text{Sin } AOC = \frac{AC}{OA} = \frac{6\frac{7}{8} \div 2}{5} = .68750;$$

therefore, $AOC = 43° \; 25' \; 57''$.

$AOB = 2 \times AOC = 2 \times 43° \; 25' \; 57''$
$= 86° \; 51' \; 54'' = 86.865°$.

By rule, Art. **137**,

$$\frac{86.865}{360} \times 78.54 = 18.95 \text{ sq. in. Ans.}$$

(**42**) By rule, Art. **129,** $A = b\,h = 7 \times 10\frac{3}{4}$ (129 inches $= 10\frac{3}{4}$ feet) $= 75\frac{1}{4}$ sq. ft. Ans.

(**43**) See Art. **130.**

Area of trapezoid $= \left(\frac{a+b}{2}\right) h = \frac{15\frac{7}{12} + 21\frac{11}{12}}{2} \times 7\frac{2}{3} = 143.75$ sq. ft. Ans.

(**44**) (*a*) Side of square having an equivalent area $= \sqrt{143.75} = 11.99$ ft. Ans.

(*b*) Diameter of circle having an equivalent area $= \sqrt{\frac{143.75}{.7854}} = \sqrt{183.0277} = 13\frac{1}{2}$ ft. Ans.

(*c*) Perimeter of square $= 4 \times 11.99 = 47.96$ ft.
Circumference of circle $= 13\frac{1}{2} \times 3.1416 = 42.41$ ft.

Difference of perimeter $= 5.55$ ft. $= 5$ ft. 6.6 in. Ans.

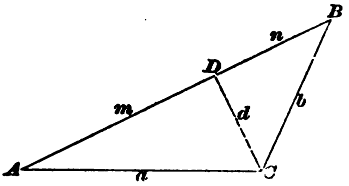

FIG. XII.

(**45**) In the triangle $A\,B\,C$, Fig. XII, $A\,B = 24$ feet, $B\,C = 11.25$ feet, and $A\,C = 18$ feet. $m + n : a + b = a - b : m - n$, or $24 : 29.25 = 6.75 : m - n$.

$$m - n = \frac{29.25 \times 6.75}{24} = 8.226562.$$

Adding $m + n$ and $m - n$, we have

$$\begin{aligned} m + n &= 24 \\ m - n &= 8.226562 \\ \hline 2\,m &= 32.226562 \\ m &= 16.113281. \end{aligned}$$

Subtracting $m - n$ from $m + n$, we have

$$\begin{aligned} 2\,n &= 15.773438 \\ n &= 7.886719. \end{aligned}$$

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In the triangle ADC, side $AC = 18$ feet, side $AD = 16.113281$; hence, according to rule **2**, Art. **98**, $\cos A = \frac{16.113281}{18} = .89518$, or angle $A = 26° 28' 5''$. In the triangle BDC, side $BD = 7.886719$, and side $BC = 11.25$ feet. Hence, $\cos B = \frac{7.886719}{11.25} = .70104$, or angle $B = 45° 29' 23''$. Angle $C = 180° - (45° 29' 23'' + 26° 28' 5'') = 108° 2' 32''$.

$$\text{Ans.} \begin{cases} A = 26° 28' 5''. \\ B = 45° 29' 23''. \\ C = 108° 2' 32'' \end{cases}$$

NOTICE

The present set of answers on the subject of Geometry and Trigonometry are less in number than were contained in the former edition. As a consequence, there is a slight break in the page numbers between the last page of the answers on Geometry and Trigonometry and the Paper following.

SURVEYING.

(QUESTIONS 614–705.)

(614) Let $x =$ number of degrees in angle C; then, $2x =$ angle A and $3x =$ angle B. The sum of A, B, and C is $x + 2x + 3x = 6x = 180°$, or $x = 30° = C$; $2x = 60° = A$, and $3x = 90° = B$. Ans.

(615) Let $x =$ number of degrees in one of the equal angles; then, $2x =$ their sum, and $2x \times 2 = 4x =$ the greater angle. $2x + 4x = 6x =$ sum of the three angles $= 180°$; hence, $x = 30°$, and the greater angle $= 30° \times 4 = 120°$. Ans.

(616) AB in Fig. 55 is the given diagonal 3.5 in. $3.5^2 = 12.25 \div 2 = 6.125$ in. $\sqrt{6.125} = 2.475$ in. = side of the required square. From A and B as centers with radii equal to 2.475 in., describe arcs intersecting at C and D. Connect the extremities A and B with the points C and D by straight lines. The figure $ACBD$ is the required square

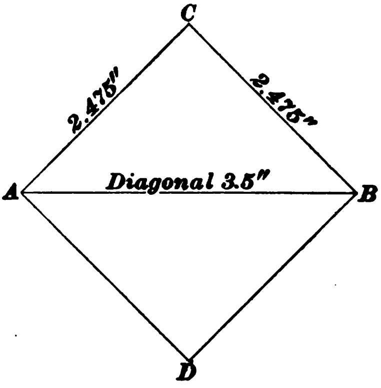

FIG. 55.

(617) Let AB, Fig. 56, be the given shorter side of the rectangle, 1.5 in. in length. At A erect an indefinite perpendicular AC to the line AB. Then, from B as a center with a radius of 3 in. describe an arc intersecting the perpendicular AC in the point D. This will give us two

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adjacent sides of the required rectangle. At B erect an indefinite perpendicular BE to AB, and at D erect an indefinite perpendicular DF to AD. These perpendiculars will intersect at G, and the resulting figure $ABGD$ will be the required rectangle. Its area is the product of the length AD by the width AB. $\overline{AD}^2 = \overline{BD}^2 - \overline{AB}^2$. $\overline{BD}^2 =$ 9 in.; $\overline{AB}^2 = 2.25$ in.; hence, $\overline{AD}^2 = 9$ in. $- 2.25$ in. $= 6.75$ in. $\sqrt{6.75} = 2.598$ in. $=$ side AD. 2.598 in. × 1.5 = 3.897 sq. in., the area of the required rectangle.

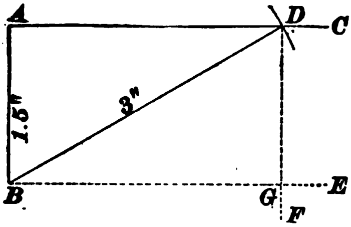

FIG. 56.

(618) (See Fig. 57.) With the two given points as centers, and a radius equal to $3.5'' \div 2 = 1.75'' = 1\frac{3}{4}''$, describe short arcs intersecting each other. With the same radius and with the point of intersection as a center, describe a circle; it will pass through the two given points.

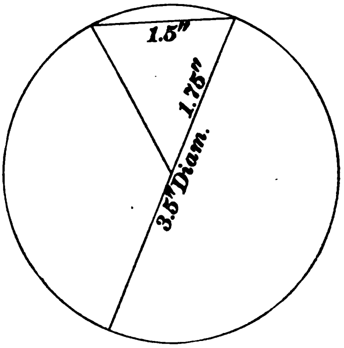

FIG. 57.

(619) AB in Fig. 58 is the given line, A the given point, AC and $CB = AB$. The angles A, B, and C are each equal to 60°. From B and C as centers with equal radii, describe arcs intersecting at D. The line AD bisects the angle A; hence, angle $BAD = 30°$.

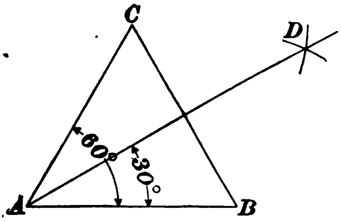

FIG. 58.

(620) Let AB in Fig. 59 be one of the given lines, whose length is 2 in., and let AC, the other line, meet AB at A, forming an angle of 30°. From A and B as centers, with radii equal to *A B*, describe arcs intersecting at *D*. Join *A D* and *B D*. The triangle *A B D* is equilateral; hence, each of its angles, as *A*, contains 60°. From *D* as a

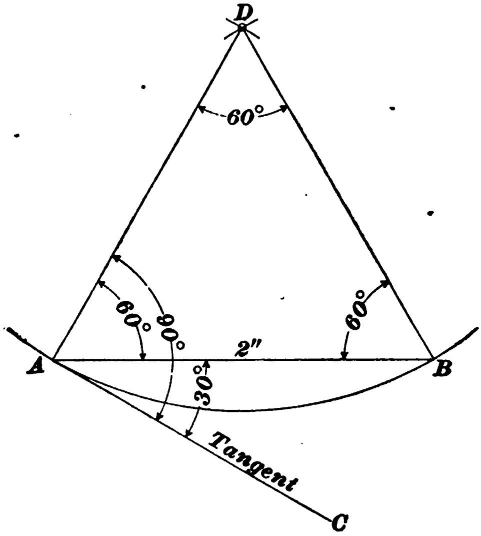

FIG. 59.

center, with a radius *A D*, describe the arc *A B*. The line *A C* is tangent to this arc at the point *A*.

(621) *A B* in Fig. 60 is the given line, *C* the given point, *C D* = *C E*. From *D* and *E* as centers with the same radii, describe arcs intersecting at *F*. Through *F* draw *C G*. Lay off *C G* and *C B* equal to each other, and from *B* and *G* as centers with equal radii describe arcs intersecting at *H*. Draw *C H*. The angle *B C H* = 45°.

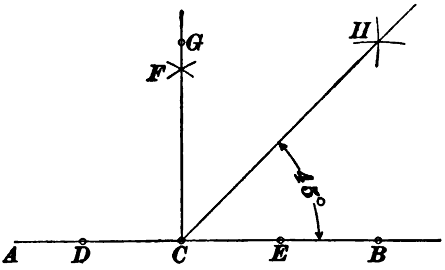

FIG. 60.

(622) 1st. $A\ B$ in Fig. 61 is the given line 3 in. long. At A and B erect perpendiculars $A\ C$ and $B\ D$ each 3 in. long. Join $A\ D$ and $B\ C$. These lines will intersect at some point E. The angles $E\ A\ B$ and $E\ B\ A$ are each 45°, and the sides $A\ E$ and $E\ B$ must be equal, and the angle $A\ E\ B = 90°$.

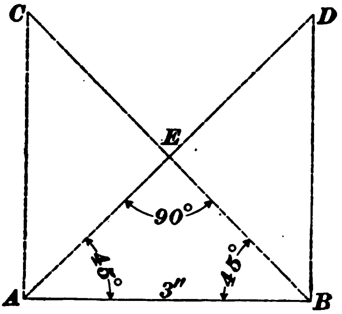

FIG. 61.

2d. $A\ B$ in Fig. 62 is the given line and c its middle point. On $A\ B$ describe the semicircle $A\ E\ B$. At c erect a perpendicular to $A\ B$, cutting the arc $A\ E\ B$ in E. Join $A\ E$ and $E\ B$. The angle $A\ E\ B$ is 90°.

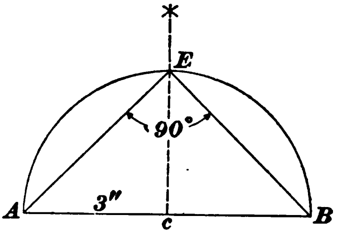

FIG. 62.

(623) See Art. **1181** and Fig. 237.

(624) See Art. **1187.**

(625) (See Art. **1195** and Fig. 245.) Draw the line $A\ B$, Fig. 63, 5 in. long, the length of the given side. At A draw the indefinite line $A\ C$, making the angle $B\ A\ C$ equal to the given angle of 30°. On $A\ C$ lay off $A\ D$ 1.5 in. long, the given difference between the other two sides of

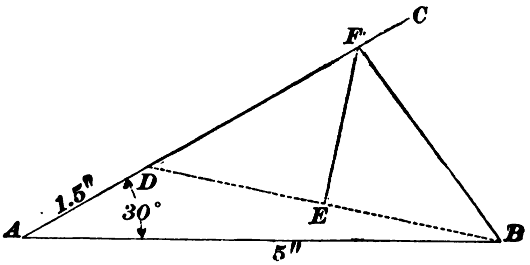

FIG. 63.

the triangle. Join the points B and D by a straight line, and at its middle point E erect a perpendicular, cutting the line $A\ C$ in the point F. Join $B\ F$. The triangle $A\ B\ F$ is the required triangle.

(626) See Art. **1198.**

(627) See Art. **1200.**

(628) See Art. **1201.**

(629) See Art. **1204.**

(630) See Art. **1205.**

(631) See Arts. **1205** and **1206.**

(632) See Art. **1206.**

(633) See Art. **1204.**

(634) See Art. **1207.**

(635) See Art. **1207.**

(636) See Arts. **1209** and **1213.**

(637) See Art. **1211.**

(638) (*a*) In this example the *declination* is *east*, and for a course whose *magnetic bearing* is N E or S W, the

Magnetic Bearing.	True Bearing.
N 15° 20′ E	N 18° 35′ E
N 88° 50′ E	S 87° 55′ E
N 20° 40′ W	N 17° 25′ W
N 50° 20′ E	N 53° 35′ E

true bearing is the *sum* of the magnetic bearing and the declination. For a course whose *magnetic bearing* is N W or S E, the *true bearing* is the difference between the magnetic bearing and the declination.

As the first magnetic bearing is N 15° 20′ E, the true bearing is the *sum* of the *magnetic bearing* and the *declination.* We accordingly make the addition as follows:

$$\begin{array}{r} \text{N } 15^\circ\ 20'\text{ E} \\ 3^\circ\ 15'\phantom{\text{ E}} \\ \hline \text{N } 18^\circ\ 35'\text{ E,} \end{array}$$

and we have N 18° 35′ E as the true bearing of the first course.

The second magnetic bearing is N 88° 50′ E, and we add the declination of 3° 15′ to that bearing, giving N 92° 05′ E. This takes us past the east point to an amount equal to the *difference* between 90° and 92° 05′, which is 2° 05′. This angle we *subtract* from 90°, the total number of degrees between the south and east points, giving us S 87° 55′ E for the *true bearing* of our line. A simpler method of determining the true bearing, when the sum of the magnetic bearing and the declination exceeds 90°, is to subtract that sum from 180°; the difference is the true bearing. Applying this method to the above example, we have 180° — 92° 05′ = S 87° 55′ E.

The third magnetic bearing is N 20° 30′ W, and the *true bearing* is the *difference* between that bearing and the *declination.* We accordingly deduct from the magnetic bearing N 20° 40′ W, the declination 3° 15′, which gives N 17° 25′ W for the true bearing.

(*b*) Here the *declination* is *west*, and for a course whose *magnetic bearing* is N W or S E the *true bearing* is the *sum* of the magnetic bearing and the declination. For a course whose *magnetic bearing* is N E or S W, the *true bearing* is the *difference* between the magnetic bearing and the declination.

The first magnetic bearing is N 7° 20′ W, and as the declination is west, it will be added. We, therefore, have for the true bearing N 7° 20′ W + 5° 10′ = N 12° 30′ W.

Magnetic Bearing.	True Bearing.
N 7° 20′ W	N 12° 30′ W
N 45° 00′ E	N 39° 50′ E
S 15° 20′ E	S 20° 30′ E
S 2° 30′ W	S 2° 40′ E

The next two bearings the student can readily determine for himself.

The fourth magnetic bearing is S 2° 30′ W, and to obtain the true bearing we must *subtract* the declination, i. e., we must change the direction eastwards. A change of 2° 30′ will bring us *due south;* hence, the bearing will be east of south to an amount equal to the difference between 2° 30′ and the total declination 5° 10′, which is 2° 40′. The true bearing is, therefore, S 2° 40′ E.

(639) See Art. **1216.**

(640) See Art. **1217.**

(641) See Art. **1219.**

(642) A plat of the accompanying notes is given in Fig. 64 to a scale of 600 ft. to the inch. The order of work is as follows:

First draw a meridian $N\ S$ (see Fig. 64), and then assume the starting point A, which call Sta. 0. Through A draw a meridian $A\ B$ parallel to $N\ S$. Then, placing the center of the protractor at A, with its zero point in the line $A\ B$, lay off the bearing angle 10° 10′ to the right of

A B, as the bearing is N E. Mark the point of angle measurement carefully, and draw a line joining it and the point *A*. This line will give the direction of the first course, the end of which is at Sta. 5 + 20, giving 520 ft. for the length of that course. On this line lay off to a scale of 600 ft. to the inch the distance 520 ft., locating the point *C*, which is Sta. 5 + 20. Through *C* draw a meridian *C D*, and with the protractor lay off the bearing angle N 40° 50′ E to the right of the meridian, marking the point of angle measurement and joining it with the point *C* by a straight line, which will be the direction of the second course. The end of this course is Sta. 10 + 89, and its length is the difference between 1,089 and 520, which is 569 ft. This distance scale off from *C*, locating the point *E* at Sta. 10 + 89. In a similar manner plat the remaining courses given in the example. Write the bearing of each line directly upon the line, taking care that the bearings shall read in the same direction as that in which the courses are being run.

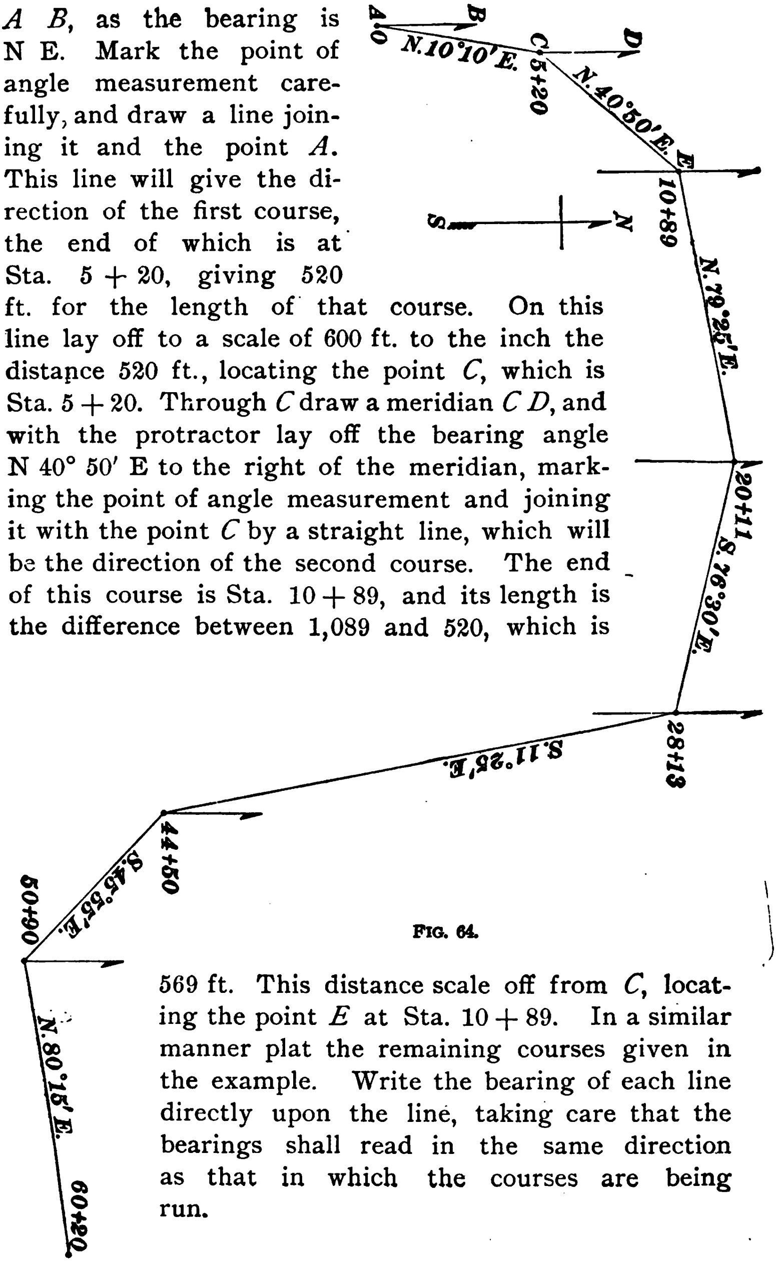

FIG. 64.

(643) See Art. **1231** and Figs. 261 and 262.

(644) For *first* adjustment see Art. **1233.** For *second* adjustment see Art. **1234** and Fig. 263, and for *third* adjustment see Art. **1235** and Fig. 264.

(645) See Art. **1238** and Fig. 265.

(646) See Art. **1239** and Fig. 266.

(647) See Art. **1240** and Fig. 267.

(648) See Art. **1242** and Fig. 269.

(649) See Art. **1242** and Fig. 269.

(650) To the bearing at the given line, viz., N 55° 15′ E, we add the angle 15° 17′, which is turned to the right. This gives for the second line a bearing of N 70° 32′ E.

(651) To the bearing of the given line, viz., N 80°11′ E, we add the angle 22° 13′, which is the amount of change in the direction of the line. The sum is 102° 24′, and the direction is 102° 24′ to the *right* or *east* of the *north* point of the compass. At 90° to the right of north the direction is due east. Consequently, the direction of the second line must be *south* of east to an amount equal to the difference between 102° 24′ and 90°, which is 12° 24′. Subtracting this angle from 90°, the angle between the south and east points, we have 77° 36′, and the direction of the second line is S 77° 36′ E. The simplest method of determining the direction of the second line is to subtract 102° 24′ from 180° 00′. The difference is 77° 36′, and the direction changing from N E to S E gives for the second line a bearing of S 77° 36′ E.

(652) To the bearing of the given line, viz., N 13° 15′ W, we add the angle 40° 20′, which is turned to the left. The sum is 53° 35′, which gives for the second line a bearing of N 53° 35′ W.

(653) The bearing of the first course, viz., S 10° 15′ W, is found in the column headed Mag. Bearing, opposite Sta. 0. For the first course, the deduced or calculated bearing must be the same as the magnetic bearing. At Sta. 4 + 40, an angle of 15° 10′ is turned to the right. It is at once evident that if a person is traveling in the direction S 10° 15′ W, and changes his course to the right 15° 10′, his course will approach a due westerly direction by the amount of the change, and the direction of his second course is found by adding to the first course, viz., S 10° 15′, the amount of such change in direction. The sum is 25° 25′, and the second course S 25° 25′ W. The needle at this point reads S 25° 20′ W. The difference between the magnetic bearing and the calculated bearing may be owing to local attraction, but as we can not read the needle to within 10 minutes, we must generally ascribe small discrepancies to that cause. This calculated bearing we write in its proper column opposite Sta. 4 + 40, where the change in direction occurred.

Station.	Deflection.	Mag. Bearing.	Ded. Bearing.
54 + 25			
49 + 20	L. 25° 14′	S 25° 40′ W	S 25° 39′ W
44 + 80	L. 10° 47′	S 50° 50′ W	S 50° 53′ W
33 + 77	R. 16° 55′	S 61° 45′ W	S 61° 40′ W
25 + 60	R. 24° 40′	S 44° 50′ W	S 44° 45′ W
16 + 20	L. 15° 35′	S 20° 00′ W	S 20° 05′ W
8 + 90	R. 10° 15′	S 35° 50′ W	S 35° 40′ W
4 + 40	R. 15° 10′	S 25° 20′ W	S 25° 25′ W
0		S 10° 15′ W	S 10° 15′ W

The next angle is 10° 15′ to the right, which we add to the previous calculated bearing S 25° 25′ W, giving S 35° 40′ W for the calculated bearing of the third course, which extends from Sta. 8 + 90 to 16 + 20. In a similar manner, the student will calculate the remaining bearings, considering well how the changes in direction will affect his relations to the points of the compass. A plat of the notes to a scale of 400 ft. to the inch is given in Fig. 65.

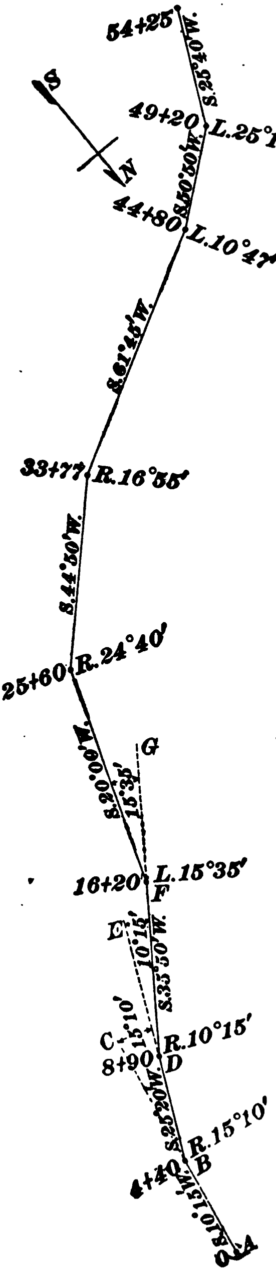

FIG. 65.

Assume the starting point *A*, Fig. 65, which is Sta. 0, and through it draw a straight line. The first angle, viz., 15° 10′ to the right, is turned at Sta. 4 + 40, giving for the first course a length of 440 ft. Scale off this distance from *0 A* to a scale of 400 ft. to the inch, locating the point *B*, which is Sta. 4 + 40, and write on the line its bearing S 10° 15′ W, as recorded in the notes. Produce *A B* to *C*, making *B C* greater than the diameter of the protractor, and from *B* lay off to the right of *B C* the angle 15° 10′. Join this point of angle measurement with *B* by a straight line, giving the direction of the second course. The end of this course is at Sta. 8 + 90, and its length is the difference between 8 + 90 and 4 + 40, which is 450 ft. This distance we scale off from *B*, Sta. 4 + 40, locating the point *D*, and write on the line its bearing of S 25° 20′ W, as found in the notes. We next produce *B D* to *E*, making *D E* greater than the diameter of the protractor, and at *D* lay off the angle 10° 15′ to the right of *D E*, giving the direction of the next course. In a similar manner, plat the remaining notes given in Example 653. The student in his drawing will show the prolongation of only the first three lines, drawing such prolongations in dotted lines. In platting the remaining angles, he will produce the lines in pencil only, erasing them as soon as the forward angle is laid off. Write the proper station number in pencil at the end of each line as soon as platted, and the angle with its direction, R. or L., before laying off the following angle. Write the bearing of each line distinctly, the letters reading in the same direction in which the line is being run. The magnetic meridian is platted as follows: The bearing of the course from Sta. 33 + 77 to Sta. 44 + 80 is S 61° 45′ W, i. e., the course is 61° 45′ to the left of a north and south line, which is the direction we wish to indicate on the map. Accordingly, we place a protractor with its center at Sta. 33 + 77 and its zero on the following course, and read off the angle 61° 45′ to the right. Through this point of angle measurement and Sta. 33 + 77 draw a straight line N S. This line is the required meridian.

(654) Angle $B = 39° 25'$. From the principles of trigonometry (see Art. **1243**), we have the following proportion:

$$\sin 39° 25' : \sin 60' 15' :: 415 \text{ ft.} : \text{side } A\,B.$$
$$\sin 60° 15' = .8682.$$
$$415 \text{ ft.} \times .8682 = 360.303 \text{ ft.}$$
$$\sin 39° 25' = .63496.$$
$$360.303 \div .63496 = 567.442 \text{ ft., the side } A\,B. \quad \textbf{Ans.}$$

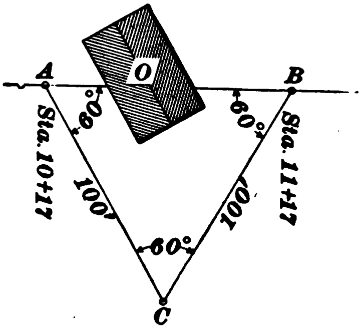

Fig. 66.

(655) (See Fig. 66.) At A we turn an angle $B\,A\,C$ of 60° and set a plug at C 100′ from A. At C we turn an angle $A\,C\,B$ of 60° and set a plug at B 100′ from C. The point B will be in the line $A\,B$, and setting up the instrument at B, we turn the angle $C\;B\;A = 60°$. The instrument is then reversed, and the line $A\,B$ produced as required.

(656) See Art. **1245.**

(657) See Art. **1246.**

(658) See Art. **1246.**

(659) See Art. **1248.**

(660) See Art. **1249.**

(661) A 5° curve is one in which a central angle of 5° will subtend a chord of 100 ft. at its circumference. Its radius is practically one-fifth of the radius of a 1° curve, and equal to 5,730 ft. $\div$ 5 = 1,146 ft.

(662) The degree of curve is always twice as great as the deflection angle for a chord of 100 feet.

(663) See Art. **1249** and Fig. 282.

(664) Formula **90,** $C = 2\,R \sin D$. (See Art. **1250.**)

(665) Formula **91,** $T = R \tan \frac{1}{2} I$. (See Art. **1251.**)

(666) The intersection angle $C\,E\,F$, being external to the triangle $A\ E\ C$, is equal to the sum of the opposite interior angles A and C. $A = 22° \ 10'$ and $C = 23° \ 15'$. Their sum is $45° \ 25' = C\,E\,F$. Ans.

The angle $A\ E\ C = 180° - (22° \ 10' + 23° \ 15') = 134° \ 35'$.

From the principles of trigonometry (see Art. **1243**), we have

$$\sin 134° \ 35' : \sin 23° \ 15' :: 253.4 \text{ ft.} : \text{side } A\ E;$$

whence, side $A\ E = 140.44$ ft., nearly. Ans.

Also, $\sin 134° \ 35' : \sin 22° \ 10' :: 253.4 \text{ ft.} : \text{side } C\,E$;

whence, side $C\,E = 134.24$ ft., nearly. Ans.

(667) We find the tangent distance T by applying formula **91,** $T = R \tan \frac{1}{2} I$. (See Art. **1251.**) From the table of Radii and Deflections we find the radius of a 6° 15′ curve = 917.19 ft.; $\frac{1}{2}\ I = \frac{35° \ 10'}{2} = 17° \ 35'$; $\tan 17° \ 35' = .3169$. Substituting these values in the formula, we have $T = 917.19 \times .3169 = 290.66$ ft. Ans.

(668) We find the tangent distance T by applying formula **91,** $T = R \tan \frac{1}{2} I$. (See Art. **1251.**) From the table of Radii and Deflections we find the radius of a 3° 15′ curve is 1,763.18 ft.; $\frac{1}{2}\ I = \frac{14^\circ\ 12'}{2} = 7^\circ\ 06'$; tan 7° 06′ = .12456. Substituting these values in the above formula, we have $T = 1{,}763 \times .12456 = 219.62$ ft. Ans.

(669) See Art. **1252.**

(670) The angle of intersection 30° 45′, reduced to decimal form, is 30.75°. The degree of curve 5° 15′, reduced to decimal form, is 5.25°. Dividing the intersection angle 30.75° by the degree of curve 5.25 (see Art. **1252**), the quotient is the required length of the curve in stations of 100 ft. each. $\frac{30.75^\circ}{5.25^\circ} = 5.8571$ full stations equal to 585.71 ft.

(671) In order to determine the P. C. of the curve, we must know the tangent distance which, subtracted from the number of the station of the intersection point, will give us the P. C. We find the tangent distance T by applying formula **91,** $T = R \tan \frac{1}{2} I$. (See Art. **1251.**) From the table of Radii and Deflections we find the radius of a 5° curve is 1,146.28 ft.; $\frac{1}{2}\ I = \frac{33^\circ\ 06'}{2} = 16^\circ\ 33'$; tan 16° 33′ = .29716. Substituting these values in formula **91,** we have $T = 1{,}146.28 \times .29716 = 340.63$ ft. In Fig. 67, let $A\ B$ and $C\ D$ be the tangents which intersect in the point E, forming an angle $D\ E\ F = 33^\circ\ 06'$. The line of survey is being run in the direction $A\ B$, and the line is measured in regular order up to the intersection point E, the station of which is 20 + 37.8. Subtracting the tangent distance, $B\ E = 340.63$ ft. from Sta. 20 + 37.8, we have 16 + 97.17, the station of the P. C. at B. The intersection angle 33° 06′ in decimal form, is 33.1°. Dividing

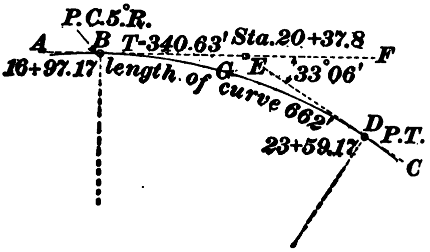

FIG. 67.

this angle by 5, the degree of the curve, we obtain the length *B G D* of the curve in full stations. $\frac{33.1}{5} = 6.62$ stations $= 662$ ft. The length of the curve, 662 ft., added to the station of the P. C., viz., $16 + 97.17$, gives $23 + 59.17$, the station of the P. T. at *D*.

(672) The given tangent distance, viz., 291.16 ft., was obtained by applying formula **91,** $T = R \tan \frac{1}{2} I$ (see Art. **1251**), $I = 20° \ 10'$, and $\frac{1}{2} I = 10° \ 05'$, $\tan 10° \ 05' = .17783$. Substituting these values in the above formula, we have $291.16 = R \times .17783$; whence, $R = \frac{291.16}{.17783} = 1{,}637.29$ ft. Ans.

The degree of curve corresponding to the radius 1,637.29 we determined by substituting the radius in formula **89,** $R = \frac{50}{\sin D}$ (see Art. **1249**), and we have

$$1{,}637.29 = \frac{50}{\sin D}; \text{ whence, } \sin D = \frac{50}{1{,}637.29} = .03054.$$

The deflection angle corresponding to the sine .03054 is 1° 45′, and is one-half the degree of the curve. The degree of curve is, therefore, $1° \ 45' \times 2 = 3° \ 30'$. Ans.

(673) Formula **92,** $d = \frac{c^2}{R}$. (See Art. **1255** and Fig. 283.)

(674) The ratio is 2; i. e., the chord deflection is double the tangent deflection. (See Art. **1254** and Fig. 283.)

(675) As the degree of the curve is 7°, the deflection angle is $3° \ 30' = 210'$ for a chord of 100 ft., and for a chord of 1 ft. the deflection angle is $\frac{210'}{100} = 2.1'$; and for a chord of 48.2 ft. the deflection angle is $48.2 \times 2.1' = 101.22' = 1° \ 41.22'$.

(676) The deflection angle for 100-ft. chord is $\frac{6° \ 15'}{2} = 3° \ 07\frac{1}{2}' = 187.5'$, and the deflection angle for a 1-ft. chord is

$\frac{187.5'}{100} = 1.875'$. The deflection angle for a chord of 72.7 ft. is, therefore, $1.875' \times 72.7 = 136.31' = 2° \ 16.31'$.

(677) We find the tangent deflection by applying formula **93,** tan def. $= \frac{c^2}{2R}$. (See Art. **1255.**) $c = 50$. $50^2 = 2,500$. The radius R of 5° 30′ curve = 1,042.14 ft. (See table of Radii and Deflections.) Substituting these values in formula **93,** we have tan def. $= \frac{2,500}{2,084.28} = 1.199$ ft. Ans.

(678) The formula for chord deflections is $d = \frac{c^2}{R}$. (See Art. **1255,** formula **92.**) $c = 35.2$. $35.2^2 = 1,239.04$. The radius R of a 4° 15′ curve is 1,348.45 ft. Substituting these values in formula **92,** we have $d = \frac{1,239.04}{1,348.45} = .919$ ft. Ans.

(679) The formula for finding the radius R is $R = \frac{50}{\sin D}$. (See Art. **1249.**) The degree of curve is 3° 10′. D, the deflection angle, is $\frac{3° \ 10'}{2} = 1° \ 35'$; $\sin 1° \ 35' = .02763$. Substituting the value of sin D in the formula, we have $R = \frac{50}{.02763}$; whence, $R = 1,809.63$ ft. Ans.

The answer given with the question, viz., 1,809.57 ft., agrees with the radius given in the table of Radii and Deflections, which was probably calculated with sine given to eight places instead of five places, as in the above calculation, which accounts for the discrepancy in results.

(680) In Fig. 68, let $A B$ and $A C$ represent the given lines, and $B C$ the amount of their divergence, viz., 18.22 ft. The lines will form a triangle $A B C$, of which the angle $A = 1°$. Draw a perpendicular from A to D, the middle point of the base. The perpendicular will

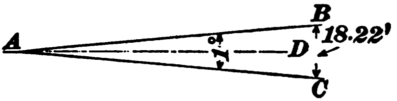

FIG. 68.

bisect the angle A and form two right angles at the base of the triangle. In the triangle ADB we have, from rule **5**, Art. **754**, $\tan BAD = \frac{BD}{AD}$. $BAD = 30'$, $BD = \frac{18.22 \text{ ft.}}{2} =$ 9.11 ft., and $\tan 30' = .00873$. Substituting known values in the equation, we have $.00873 = \frac{9.11}{AD}$; whence, $AD = \frac{9.11 \text{ ft.}}{.00873} = 1{,}043.53$ ft. Ans.

By a practical method, we determine the length of the lines by the following proportion:

$$1.745 : 18.22 :: 100 \text{ ft.} : \text{the required length of line};$$

$$\text{whence, length of line} = \frac{1{,}822}{1.745} = 1{,}044.13 \text{ ft.} \quad \text{Ans.}$$

The second result is an application of the principle of two lines 100 ft. in length forming an angle of 1° with each other, which will at their extremity diverge 1.745 ft.

(681) Degree of curve $= \frac{24° \, 15'}{6.0625} = \frac{24.25°}{6.0625} = 4°$. Ans.

(682) See Arts. **1264, 1265, 1266,** and **1267.**

(683) See Arts. **1269-1274.**

(684) Denote the radius of the bubble tube by x; the distance of the rod from the instrument, viz., 300', by d; the difference of rod readings, .03 ft., by h, and the movement of the bubble, viz., .01 ft., by S. By reference to Art. **1275** and Fig. 289, we will find that the above values have the proportion $h : S :: d : x$. Substituting known values in the proportion, we have $.03 : .01 :: 300 : x$; whence, $x = \frac{3}{.03} = 100$ ft., the required radius. Ans.

(685) See Art. **1277.**

(686) See Art. **1278.**

(687) To the elevation 61.84 ft. of the given point, we add 11.81 ft., the backsight. Their sum, 73.65 ft., is the height of instrument. From this H. I., we subtract the foresight to the T. P., viz., 0.49 ft., leaving a difference of 73.16 ft., which is the elevation of the T. P. (See Art. **1279.**)

(688) See Art. **1280.**

(689) See Art. **1281.**

(690) See Art. **1282.**

(691) See Art. **1286.**

(692) See Art. **1289.**

(693) See Art. **1290.**

(694) See Art. **1291.**

(695) The distance between Sta. 66 and Sta. 93 is 27 stations. As the rate of grade is + 1.25 ft. per station, the total rise in the given distance is 1.25 ft. × 27 = 33.75 ft., which we add to 126.5 ft., the grade at Sta. 66, giving 160.25 ft. for the grade at Sta. 93. (See Art. **1291.**)

(696) See Art. **1292.**

(697) $\frac{-16.4'}{56'}$ $\frac{-10.3'}{73'}$	Center Line.	$\frac{+11.4'}{84'}$ $\frac{+8.8'}{96'}$
Contour 50.0 at 46.0 ft. to left of Center Line.		Contour 60.0 at 26 ft. to right of Center Line.
Contour 40.0 at 94.0 ft. to left of Center Line.		Contour 70.0 at 106.9 ft. to right of Center Line.
Contour 30.0 at 128.0 ft. to left of Center Line.		Elevation 76.7 at 180 ft. to right of Center Line.

(698) The elevations of the accompanying level notes are worked out as follows: The first elevation recorded in the column of elevations is that of the *bench mark*, abbreviated to B. M. This elevation is 161.42 ft. The first rod reading, 5.53 ft., is the backsight on this B. M., a *plus* reading, and recorded in column of rod readings. This rod reading we add to the elevation of the bench mark, to determine the height of instrument, as follows: 161.42 ft. + 5.53 ft. = 166.95 ft., the H. I. The next rod reading, which is at Sta. 40, is 6.4 ft. The rod reading means that the surface of the ground at Sta. 40 is 6.4 ft. below the horizontal axis of the telescope. The elevation of that surface is, therefore, the difference between 166.95 ft., the H. I., and 6.4 ft., the rod reading. 166.95 − 6.4 = 160.55 ft. The $\frac{5}{100}$ ft. is a fraction so small that in surface elevations it is the universal practice to ignore it, and the elevation of the ground at

Station.	Rod Reading.	Height Instrument.	Elevation.	Grade.
B. M.	+ 5.53	166.95	161.42	
40	6.4		160.5	162.0
41	7.2		159.7	160.485
41 + 60	10.9		156.0	
42	8.6		158.3	158.97
43	8.8		158.1	157.455
T. P. –	8.66		158.29	
+	2.22	160.51		
44	4.8		155.7	155.94
45	6.3		154.2	154.425
46	8.8		151.7	152.91
47	9.9		150.6	151.395
48	11.1		149.4	149.88
T. P. –	11.24		149.27	
+	3.30	152.57		
49	4.7		147.9	148.365
50	7.1		145.5	146.85
51	8.7		143.9	145.335
52	9.8		142.8	143.82
53	10.9		141.7	142.305
T. P. –	11.62		140.95	

Sta. 40 is taken at 160.5 ft. The rod reading at Sta. 41 is 7.2, which, subtracted from 166.95 ft., gives for that station an elevation of 159.7. The remaining rod readings up to and including that at Sta. 43, we subtract from the same H. I., viz., 166.95. Here at a T. P., a rod reading of 8.66 ft. is taken and recorded in the column of rod readings. This reading being a foresight is *minus*, and is subtracted from the preceding H. I. This gives us for the elevation of the T. P., 166.95 ft. — 8.66 ft. = 158.29 ft., which we record in the column of elevations. The instrument is then moved forwards and a backsight of 2.22 ft. taken on the same T. P. and recorded in the column. This is a *plus* reading, and is added to the elevation of the T. P., giving us for the next H. I. an elevation of 158.29 ft. + 2.22 ft. = 160.51 ft. The next rod reading, viz., 4.8, is at Sta. 44, and the elevation at that station is the difference between the preceding H. I., 160.51, and that rod reading, giving an elevation of 160.51 ft. — 4.8 ft. = 155.7 ft., which is recorded in the column of elevations opposite Sta. 44. In a similar manner, the remaining elevations are determined.

In checking level notes, only the *turning points* rod readings are considered. It will be evident that starting from a given bench mark, all the *backsight* or *plus* readings will add to that elevation, and all the *foresight* or *minus* readings will subtract from that elevation. If now we place in one column the height of the B. M., together with all the backsight or + readings, and in another column all the foresight or — readings, and find the sum of each column, then, by subtracting the sum of the — readings from the sum of the + readings, we shall find the elevation of the last point calculated, whether it be a turning point or a height of instrument. Applying this method to the foregoing notes, we have the following:

	+ readings.	— readings.
B. M.	161.42 ft.	8.66 ft.
	5.53 ft.	11.24 ft.
	2.22 ft.	11.62 ft.
	3.30 ft.	31.52 ft.
	172.47 ft.	
	31.52 ft.	
	140.95 ft.	

The difference of the columns, viz., 140.95 ft., agrees with the elevation of the T. P. following Sta. 53, which is the last one determined. A check mark √ is placed opposite the elevation checked, to show that the figures have been verified. The rate of grade is determined as follows: In one mile there are 5,280 ft. = 52.8 stations. A descending grade of 80 ft. per mile gives per station a descent of $\frac{80 \text{ ft.}}{52.8} = 1.515$ ft. The elevation of the grade at Sta. 40 is fixed at 162.0 ft. As the grade descends from Sta. 40 at the rate of 1.515 ft. per station, the grade at Sta. 41 is found by subtracting 1.515 ft. from 162.0 ft., which gives 160.485 ft., and the grade for each succeeding station is found by subtracting the rate of grade from the grade of the immediately preceding station.

A section of profile paper is given in Fig. 69 in which the level notes are platted, and upon which the given grade line is drawn. The profile is made to the following scales: viz., horizontal, 400 ft. = 1 in.; vertical, 20 ft. = 1 in.

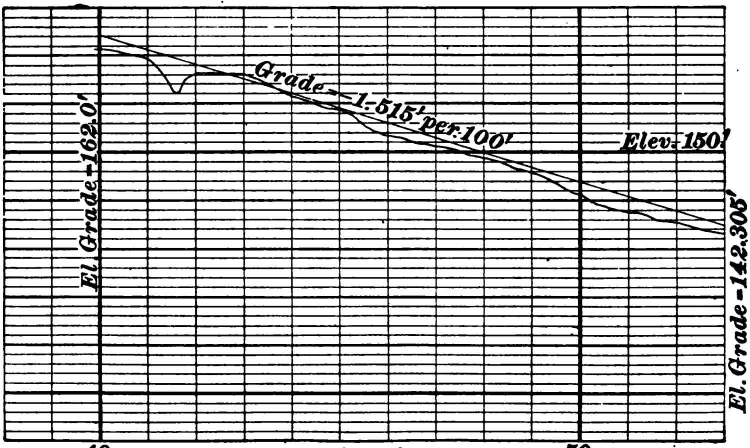

FIG. 69.

Every fifth horizontal line is heavier than the rest, and each twenty-fifth horizontal line is of *double weight*. Every tenth vertical line is of *double weight*. The spaces between the vertical lines represent 100 ft., and those between the horizontal lines 1 ft. The figure represents 1,500 ft. in length and 45 ft. in height. Assume the elevation of the sixth heavy line from the bottom at 150 ft. The second vertical line from the left of the figure is Sta. 40, which is written in the margin at the bottom of the page. Under the next heavy vertical line, ten spaces to the right, Sta. 50 is written. The elevation of Sta. 40, as recorded in the notes, is 160.5 ft. We determine the corresponding elevation in the profile as follows:

As the elevation of the sixth heavy line from the bottom is assumed at 150 ft., 160.5 ft., which is 10.5 ft. higher, must be 10½ spaces above this line. This additional space covers two heavy lines and one-half the next space. This point is marked in pencil. The elevation of Sta. 41, viz., 159.7 ft., we locate on the next vertical line and 9.7 spaces above the 150 ft. line. The next elevation, 156.0 ft., is at Sta. 41 + 60. This distance of 60 ft. from Sta. 41 we estimate by the eye and plat the elevation in its proper place. In a similar manner, we plat the remaining elevations and connect the points of elevation by a continuous line drawn free-hand. The grade at Sta. 40 is 162.0 ft. This elevation should be marked in the profile by a point enclosed by a small circle. At each station between Sta. 40 and Sta. 53 there has been a descent of 1.515 ft., making a total descent between these stations of $1.515 \times 13 = 19.695$. The grade at Sta. 53 will, therefore, be 162.0 ft. − 19.695 ft. = 142.305 ft. Plat the elevation in the profile at Sta. 53, and enclose the point in a small circle. Join the grade point at Sta. 40 with that at Sta. 53 by a straight line, which will be the grade line required. Upon this line mark the grade − 1.515 per 100 ft.

(699) $\frac{-5^\circ}{65'} \qquad \frac{-9^\circ}{117'} \quad \Bigg| \quad \frac{+11^\circ}{120'}$

Nine 5-foot contours are included within the given slopes, as follows:

Contour 70.0 at 32.0 ft. to left of Center Line.	Center Line.	Contour 80.0 at 26.0 ft. to right of Center Line.
Contour 65.0 at 64.0 ft. to left of Center Line.		Contour 85.0 at 52.0 ft. to right of Center Line.
Contour 60.0 at 96.0 ft. to left of Center Line.		Contour 90.0 at 78.0 ft. to right of Center Line.
Contour 55.0 at 136.0 ft. to left of Center Line.		Contour 95.0 at 104.0 ft. to right of Center Line.
Elevation 51.0 at 182.0 ft. to left of Center Line.		Elevation 98.1 at 120.0 ft. to right of Center Line.

(700) 1.745 ft. × 3 = 5.235 ft., the vertical rise of a 3° slope in 100 ft., or 1 station and $\frac{10}{5.235} = 1.91$ stations = 191 ft.

(701) (See Question 701, Fig. 15.) From the instrument to the center of the spire is 100 ft. + 15 = 115 ft., and we have a right triangle whose base $A\,D = 115$ ft. and angle A is 45° 20′. From rule **5,** Art. **754,** we have tan 45° 20′ = $\frac{\text{side } B\,D}{115}$; whence, $1.01170 = \frac{\text{side } B\,D}{115}$; or $B\,D = 116.345$ feet. The instrument is 5 feet above the level of the base; hence, 116.345 ft. + 5 ft. = 121.345, the height of the spire.

(702) Apply formula **96.**

$$Z = (\log h - \log H) \times 60{,}384.3 \times \left(1 + \frac{t + t' - 64^\circ}{900}\right),$$

(See Art. **1304.**)

$$\begin{aligned} \text{log of } h,\ 29.40 &= 1.46835 \\ \text{log of } H,\ 26.95 &= \underline{1.43056} \\ \text{Difference} &= 0.03779 \end{aligned}$$

$$1 + \frac{t + t' - 64^\circ}{900} = 1 + \frac{74 + 58 - 64}{900} = 1.0755.$$

Hence, $Z = .03779 \times 60{,}384.3 \times 1.0755 = 2{,}454$ ft., the difference in elevation between the stations.

(703) See Art. **1305.**

(704) See Art. **1308.**

(705) See Art. **1308.**

LAND SURVEYING.

(QUESTIONS 706–755.)

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(706) See Art. **1309.**

(707) See Art. **1309.**

(708) See Art. **1310.**

(709) See Art. **1312.**

(710) See Art. **1313.**

(711) See Art. **1310** and Fig. 306.

(712) See Art. **1314** and Fig. 308.

(713) See Art. **1314** and Fig. 309.

(714) See Art. **1314.**

(715) See Art. **1315** and Fig. 310.

(716) See Art. **1317.**

(717) See Art. **1318.**

(718) See Art. **1319.**

(719) See Art. **1319** and Figs. 311 and 312.

(720) See Art. **1319.**

(721) See Art. **1319.**

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(722) See Art. **1319.**

(723) See Art. **1320.**

(724) See Art. **1320.**

(725) See Art. **1321.**

(726) The magnetic variation is determined by subtracting the present bearing from *B* to *C*, viz., N 60° 15′ E from the original bearing, viz., N 62° 00′ E. The difference is 1° 45′ and an *east* variation; hence, to determine the present bearings of the boundaries we must *add* the *variation* to an original bearing, which was N W or S E, and *subtract* it from an original bearing, which was N E or S W. The corrected bearings will be as follows:

Stations.	Original Bearings.	Distances.	Corrected Bearings.
A	N 31½° W	10.4 chains	N 33¼° W
B	N 62° E	9.2 chains	N 60° 15′ E
C	S 36° E	7.6 chains	S 37¾° E
D	S 45½° W	10.0 chains	S 43¾° W

(727) See Art. **1323.**

(728) See Art. **1324.**

(729) See Art. **1326** and Fig. 316.

(730) See Art. **1327** and Figs. 317 and 318.

(731) See Art. **1328** and Fig. 319.

(732) As the bearing of the line *A B* is N E, the end *B* will be east of the meridian passing through *A*. The departure of AB is the distance which B is east of A, or of the meridian passing through A. Now, if from B we drop a perpendicular BC upon that meridian, BC will be the departure of AB. The latitude of AB is the distance which the end B is north of the end A. The distance AC, measured on the meridian from A to the foot of the perpendicular from B, is the latitude of AB.

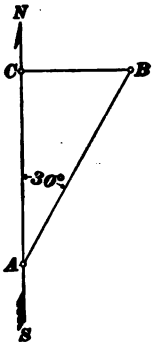

FIG. 70.

From an inspection of Fig. 70, we see that the line AB, together with its latitude AC and departure BC, form a right triangle, right angled at C, of which triangle BC is the sin and AC the cos of the bearing 30°. From rule **3**, Art. **754**, we have $\cos A = \frac{AC}{AB}$; whence, $AC = AB \cos A$; and from rule **1**, Art. **754**, $\sin A = \frac{BC}{AB}$; whence, $BC = AB \sin A$, and we deduce the following:

Latitude = distance × cos bearing.
Departure = distance × sin bearing.

(733) See Art. **1329.**

(734)

Bearing.	Distances.	Latitudes.	Departures.
23¼°	400 ft.	3675	1579
	20 ft.	1838	0789
	3 ft.	2756	1184
	423 ft.	388.636 ft.	166.974 ft.

We divide the distance 423 ft. into three parts, viz., 400 ft., 20 ft., and 3 ft. If now we find the latitude and departure for 4 ft. and multiply them by 100, we shall obtain the latitude and departure for 400 ft. The latitude of 4 ft. is 3.675 ft., and the departure 1.579 ft. We place these figures under their proper headings as whole numbers. The latitude and departure of 2 ft. are 1.838 and 0.789, respectively, which we place as whole numbers under their proper headings, but removed one place to the right of the figures above them, as they are the latitude and departure of tens of feet. The latitude and departure of 3 ft. are 2.756 ft. and 1.184 ft., respectively, and we place them under their proper headings, but removed one place to the right as they are for units of feet. We now add up the partial latitudes and departures, and from the right of each sum we point off three decimal places, the same number as given in the traverse table, giving us for for the required latitude 388.636 ft., and for the required departure 166.974 ft.

(735)

Bearing.	Distances.	Latitudes.	Departures.
40°	200 ft.	1532	1286
	20 ft.	1532	1286
	5 ft.	3830	3214
	225 ft.	172.350 ft.	144.674 ft.

For the given bearing of 40° and distance of 225 ft., the latitude is 172.35 ft., and the *departure* 144.674 ft.

The complement of the given bearing is the difference between 90° and 40°, which is 50°. With this complement as the bearing, we have

Bearing.	Distances.	Latitudes.	Departures.
50°	200 ft.	1286	1532
	20 ft.	1286	1532
	5 ft.	3214	3830
	225 ft.	144.674 ft.	172.350 ft.,

in which the latitude and departure are exactly the reverse of those when the line had a bearing of 40°, the complement of 50°.

(736) See Art. **1330.**

(737) We rule 11 columns, headed as below. The latitudes and departures for the several courses we calculate by traverse tables; placing the *north* latitudes, which are +, in the column headed N +, and the *south* latitudes, which are —, in the column headed S — ; the *east* departures, which are

+, in the column headed E +, and the *west* departures, which are —, in the column headed W —. These several columns we add, placing their sums at the foot of the columns. The sum of the distances is 37.20 chains; the sum of the north latitudes 13.19 chains, and of the south latitudes 13.16 chains. The difference is .03 chain, or 3 links. The sum of the east departures is 12.60 chains, and the sum of the west departures is 12.56 chains. The difference is .04 chain, or 4 links.

This difference indicates an error in either the bearings or measurements of the line or both. For had the work been correct, the sums of the north and south latitudes would have been equal. (See Art. **1330.**) The corrections for latitudes and departures are made as shown in the following proportions, the object of such correction being to make the sums of the north and south latitudes and of the east and west longitudes equal, and is called balancing the survey. (See Art. **1331.**)

Stations.	Bearings.	Distances.	Latitudes.		Departures.		Corrected Latitudes.		Corrected Departures.	
			N +	S −	E +	W −	N +	S −	E +	W −
1	N 31½° W	10.40 ch.	8.87			5.43	8.86			5.44
2	N 62° E	9.20 ch.	4.32		8.13		4.31		8.12	
3	S 36° E	7.60 ch.		6.15	4.47			6.15	4.46	
4	S 45½° W	10.00 ch.		7.01		7.13		7.02		7.14
		37.20	13.19	13.16	12.60	12.56	13.17	13.17	12.58	12.58

Difference between N and S latitudes = .03 chain = 3 links.

Difference between E and W departures = .04 chain = 4 links.

Corrections for Latitudes.	Corrections for Departures.
37.20 : 10.40 :: 3 : 1 link	37.20 : 10.40 :: 4 : 1 link
37.20 : 9.20 :: 3 : 1 link	37.20 : 9.20 :: 4 : 1 link
37.20 : 7.60 :: 3 : 0 link	37.20 : 7.60 :: 4 : 1 link
37.20 : 10 00 :: 3 : 1 link	37.20 : 10.00 :: 4 : 1 link

Taking the first proportion, we have 37.20 ch., the sum of all the distances : 10.40 ch., the first distance :: 3 links, the total error : 1 link, the correction for the first distance. The latitude of the first course, viz., 8.87 ch., is north, and as the sum of the north latitudes is the greater, we *subtract* the correction leaving 8.86 chains. The correction for the latitudes of the second course is 1 link, and is likewise subtracted. The correction for the third course is less than 1 link, and is ignored. The correction for the latitude of the fourth course is 1 link, and as the sum of the south latitudes is less than the north latitudes we *add* the correction. We place the corrected latitudes in the eighth and ninth columns. In a similar manner we correct the departures, as shown in the above proportions, placing the corrected departures in the tenth and eleventh columns.

(738) We rule three columns as shown below, the first column for stations, the second for total latitudes from Sta. 2, and the third for total departures from Sta. 2. Station 2 being a point only, its latitude and departure are 0. The latitude of the second course, i. e., from Sta. 2 to Sta. 3, is + 4.31 chains, and the departure + 8.12 chains. These distances we place opposite Sta. 3, in their proper columns. The latitude of the third course, i. e., from Sta. 3 to Sta. 4, is — 6.15 chains, and the departure + 4.46 chains. Therefore, the total latitude from Sta. 2 is the sum of + 4.31 and — 6.15, which is — 1.84 chains. The total departure from Sta. 2 is the sum of + 8.12 and + 4.46, which is + 12.58 chains. These totals we place opposite Sta. 4 in their proper columns. The latitude of the fourth course, i. e., from Sta. 4 to Sta. 1, is — 7.02 chains, and the departure — 7.14 chains. These quantities we add with their proper signs to those previously obtained, which give us the total latitudes and departures from the initial Sta. 2, and we have, for the total latitude of Sta. 1, the sum of — 1.84 and — 7.02, which is — 8.86 chains, and for the total departure the sum of + 12.58 and — 7.14, which is + 5.44 chains. The latitude of the first course, i. e., from Sta. 1 to Sta. 2, is + 8.86 chains, and the departure − 5.44 chains. These quantities we add with their proper signs to those already obtained, giving us the total latitudes and departures from Sta. 2, and we have for the total latitude of Sta. 2 the sum of − 8.86 and + 8.86, which is 0; and for the total departure, the sum of + 5.44 and − 5.44, which is 0. The latitude and departure of Sta. 2 coming out equal to 0, proves the work to be correct. (See Art. **1332.**) A plat of this survey made from total latitudes and departures from Sta. 2 is given in Fig. 71. Through Sta. 2 draw a meridian *N S*. Lay off on this meridian above Sta. 2 the total latitude at Sta. 3, viz., + 4.31 chains, a north latitude, and at its extremity erect a right perpendicular to the meridian, and upon this perpendicular scale off the total departure of Sta. 3, viz., + 8.12 chains, an east departure locating Sta. 3. A line joining Stations 2 and 3 will have the direction and length of the second course. For Sta. 4 we have a total latitude of − 1.84 chains, a south latitude which we scale off on the meridian below Sta. 2. The total departure of this station is + 12.58 chains, which we lay off on a right perpendicular to the meridian, locating Sta. 4. A line joining Stations 3 and 4 gives the direction and length of the third course.

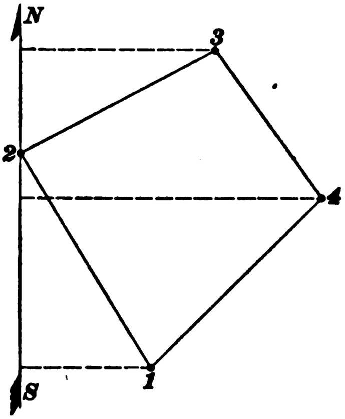

FIG. 71.

Stations.	Total Latitudes from Station 2.	Total Departures from Station 2.
2	0.00 ch.	0.00 ch.
3	+ 4.31 ch.	+ 8.12 ch.
4	− 1.84 ch.	+ 12.58 ch.
1	− 8.86 ch.	+ 5.44 ch.
2	0.00 ch.	0.00 ch.

The total latitude of Sta. 1 is — 8.86 chains, a south latitude, which we scale off on the meridian below Sta. 2. The total departure of Sta. 1 is + 5.44 chains, an east departure, which we scale off on a right perpendicular to the meridian, locating Sta. 1. A line joining Stations 4 and 1 will have the direction and length of the fourth course. The total latitude and departure of Sta. 2 being 0, a line joining Sta. 1 with Sta. 2 will have the direction and length of the first course, and the resulting figure 2, 3, 4, 1 is the required plat of the survey.

(739) See Art. **1334.**

(740) See Art. **1335.**

(741) See Art. **1336** and Fig. 323.

(742) See Art. **1336.**

(743) The signs of the latitude always determine the character of the products, north or + latitudes giving north products and south or — latitudes giving south products. See Examples 746 and 747.

(744) See Art. **1338.**

(745) Rule twelve columns with headings as shown in the following diagram, calculate the latitudes and departures, placing them in proper order. Balance them, writing the corrected latitudes directly above the original latitudes, which are crossed out. Calculate the double longitudes from Sta. 2 by rule given in Art. **1335,** and place them with their corresponding stations as shown in columns 11 and 12. Place the double longitudes in regular order in column 8. Multiply the double longitude of each course by the corrected latitude of that course, placing the products in column 9 or 10, according as the products are north or south. Add the columns of double areas, subtracting the less from the greater and divide the remainder by 2. In this example the area is given in square chains, which we reduce to acres, roods, and poles as follows: Divide the sq. chains by 10, reducing to acres. Multiply the decimal part successively by 4 and 40, reducing to roods and poles.

Stations.	Bearings.	Distances.	Latitudes.		Departures.		Double Longitudes.	Double Areas.	
			N +	S −	E +	W −		N +	S −
				11.62					
1	S 21° W	12.41 ch.		~~11.59~~		4.44	+ 4.44		51.5928
2	N 83¼° E	5.86 ch.	0.69		5.82		+ 5.82	4.0158	
			8.05						
3	N 12° E	8.25 ch.	~~8.07~~		1.72		+13.36	107.5480	
			2.88						
4	N 47° W	4.24 ch.	~~2.89~~			3.10	+11.98	34.5024	
		30.76 ch.	~~11.65~~	~~11.59~~	7.54	7.54		146.0662	
			11.62	11.62	7.54	7.54		51.5928	

Stations.	D. L.
2	+ 5.82 D. L.
	+ 5.82
	+ 1.72
3	+ 13.36 D. L.
	+ 1.72
	− 3.10
4	+ 11.98 D. L.
	− 3.10
	− 4.44
1	+ 4.44 D. L.

2) 94.4734
10) 47.2367 sq. ch.
4.72367
4
2.89468
40
35.78720

Content = 4 A. 2 R. 35.8 P.

In this example, the meridian from which we calculate the double longitudes passes through Sta. 2, the extreme westerly one.

(746)

Stations.	Bearings.	Distances.	Latitudes.		Departures.		Double Longitudes.	Double Areas.	
			N +	S −	E +	W −		N +	S −
				16.41		6.40			
1	S 21¼° W	17.62 ch.		~~16.42~~		~~6.38~~	− 6.40		105.0240
						5.60			
2	S 34° W	10.00 ch.		8.29		~~5.59~~	− 18.40		152.5360
			7.92			11.76			
3	N 56° W	14.15 ch.	~~7.91~~			~~11.73~~	− 35.76	283.2192	
					5.45				
4	N 34° E	9.76 ch.	8.09		~~5.46~~		− 42.07	340.3463	
5	N 67° E	2.30 ch.	0.90		2.12		− 34.50	31.0500	
6	N 23° E	7.03 ch.	6.47		2.75		− 29.63	191.7061	
7	N 18½° E	4.43 ch.	4.20		1.40		− 25.48	107.0160	
				2.88	12.04				
8	S 76½° E	12.41 ch.		~~2.89~~	~~12.06~~		− 12.04		34.6752
		77.70 ch.	~~27.57~~	~~27.60~~	~~23.79~~	~~23.70~~		953.3376	292.2352
			27.58	27.58	23.76	23.76		292.2352	
								2)661.1024	
								10)330.5512 sq. ch.	
								33.05512	
								4	
								.22048	
								40	
								8.81920	

Content = 33 A. 0 R. 8.8 P.

In this example, the meridian from which we calculate the double longitudes passes through Sta. 1, the extreme easterly one.

Stations.	D. L.
1	− 6.40 D. L − 6.40 − 5.60
2	− 18.40 D. L. − 5.60 − 11.76
3	− 35.76 D. L. − 11.76 + 5.45
4	− 42.07 D. L. + 5.45 + 2.12
5	− 34.50 D. L. + 2.12 + 2.75
6	− 29.63 D. L. + 2.75 + 1.40
7	− 25.48 D. L. + 1.40 + 12.04
8	− 12.04 D. L.

(747)

Stations.	Bearings.	Distances. Chains.	Latitudes.		Departures.		Double Longitudes.	Double Areas.	
			N +	S −	E +	W −		N +	S −
1	N 18¾° E	1.93	1.83		0.62		+ 1.10	2.0130	
2	N 9° W	1.29	1.27			0.20	+ 1.52	1.9304	
3	N 14° W	2.71	2.63			0.66	+ 0.66	1.7358	
4	N 74° E	0.95	0.26		0.91		+ 0.91	0.2366	
5	S 48½° E	1.59		1.05	1.19		+ 3.01		3.1605
6	S 14¼° E	1.14		1.10	0.28		+ 4.48		4.9280
7	S 19½° E	2.15		2.03	0.72		+ 5.48		11.1244
8	S 23½° W	1.22		1.12		0.49	+ 5.71		6.3952
9	S 5° W	1.40		1.39		0.12	+ 5.10		7.0890
10	S 30° W	1.02		0.88		0.51	+ 4.47		3.9336
11	S 81½° W	0.69		0.10		0.68	+ 3.28		0.3280
12	N 32½° W	1.98	1.67			1.06	+ 1.54	2.5718	
			7.66	7.67	3.72	3.72		8.4876	36.9587 8.4876

2)28.4711

Content, 1 A. 1 R. 27¾ P.

Sq. chains 10)14.2355

1.42355

4

1.69420

40

27.76800

Stations.	D. L.
4	+ 0.91 D.L. + 0.91 + 1.19
5	+ 3.01 D.L. + 1.19 + 0.28
6	+ 4.48 D.L. + 0.28 + 0.72
7	+ 5.48 D.L. + 0.72 − 0.49
8	+ 5.71 D.L. − 0.49 − 0.12
9	+ 5.10 D.L. − 0.12 − 0.51
10	+ 4.47 D.L. − 0.51 − 0.68
11	+ 3.28 D.L. − 0.68 − 1.06
12	+ 1.54 D.L. − 1.06 + 0.62
1	+ 1.10 D.L. + 0.62 − 0.20
2	+ 1.52 D.L. − 0.20 − 0.66
3	+ 0.66 D.L.

In this example, the meridian from which we calculate the double longitudes passes through Sta. 4, the extreme westerly one.

(748) See Art. **1339.**

(749) See Art. **1340** and Fig. 326.

(750) See Art. **1341** and Fig. 327.

(751) See Art. **1341.**

(752) See Art. **1342.**

(753) See Art. **1343** and Fig. 327.

(754) See Art. **1344.**

(755) See Art. **1343.**

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